Ultrafast Digital Electronics: Optimizing the speed of a Josephson junction

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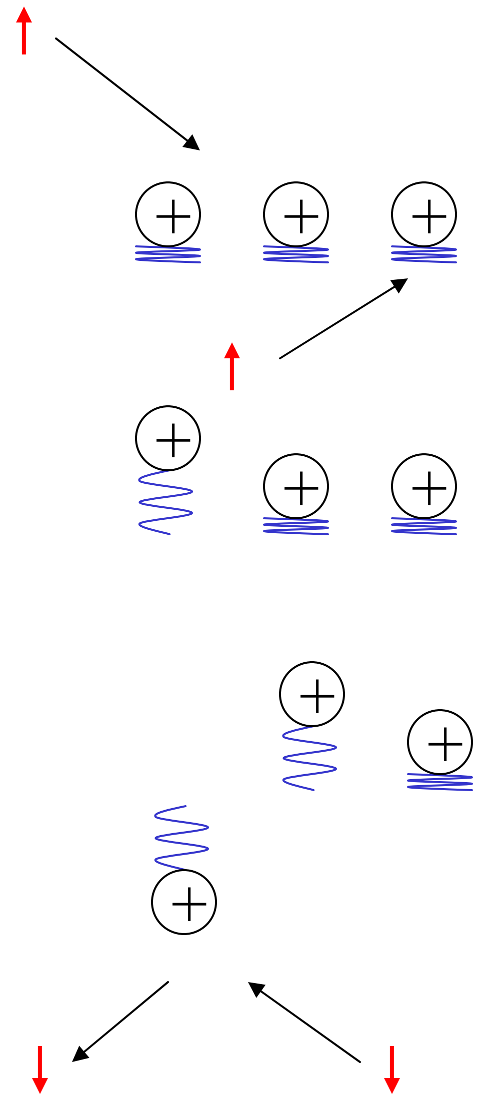
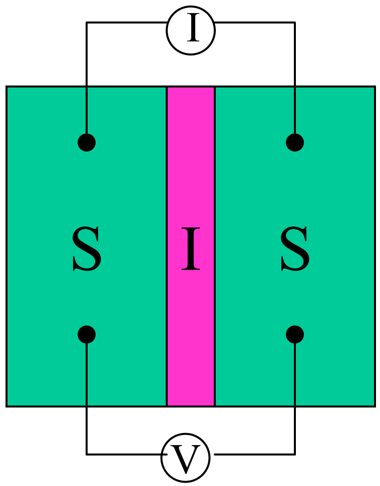
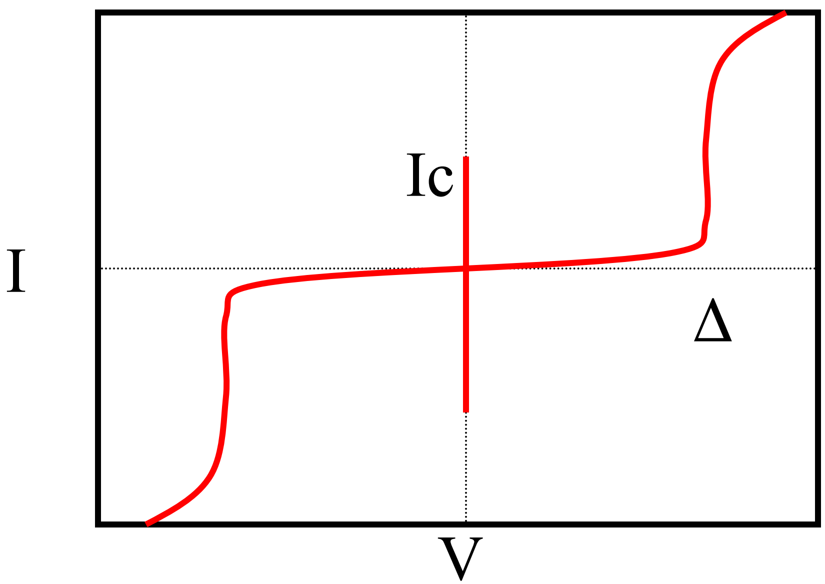
Introduction to Superconductivity

- In conventional superconductors electrons pair through a ***retarded*** interaction with the lattice vibrations (phonons).
- Superconductivity can be described by the generalization of Bardeen, Cooper, and Schrieffer's theory to include strong interactions and retardation as formulated by Migdal and Eliashberg
- Tc's tend to be low < 25 K and the order parameter is spatially symmetric (examples include Nb, Pb, Al, Hg, In, etc.).
- High Tc superconductors with Tc's as large as 150 K arise from a different mechanism (most likely driven by electron correlations) and have a d-wave spatial symmetry (examples include YBCO and BSCCO).



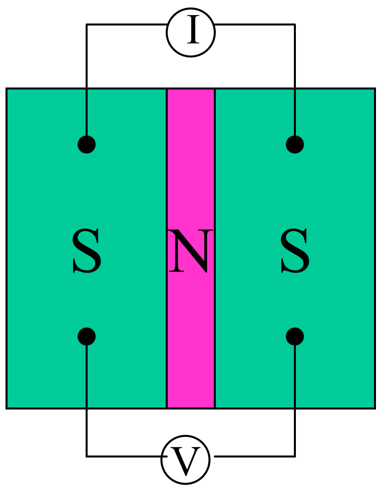
Josephson Tunnel Junctions

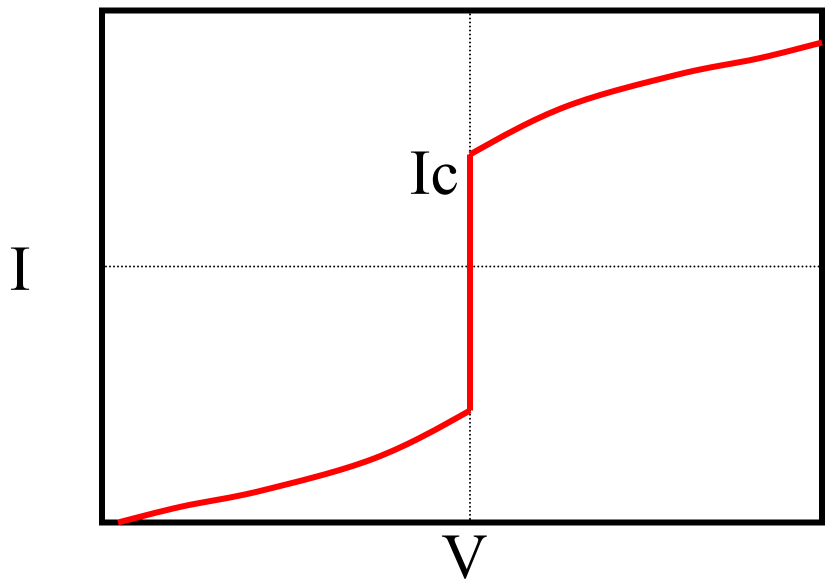




- A Superconductor-Insulator-Superconductor sandwich can tunnel coherent Cooper pairs (Josephson current) or can tunnel broken pairs (quasiparticles) through the barrier.
- If the phases of the superconducting wavefunctions differ, then there is a DC Josephson current $I = I_c \sin\theta$.
- The I-V characteristic is highly nonlinear at low voltages, leading to the possibility of important electronics applications (based on latching technologies which are slow and subject to “punch-through”).

Josephson Proximity-Effect Junctions

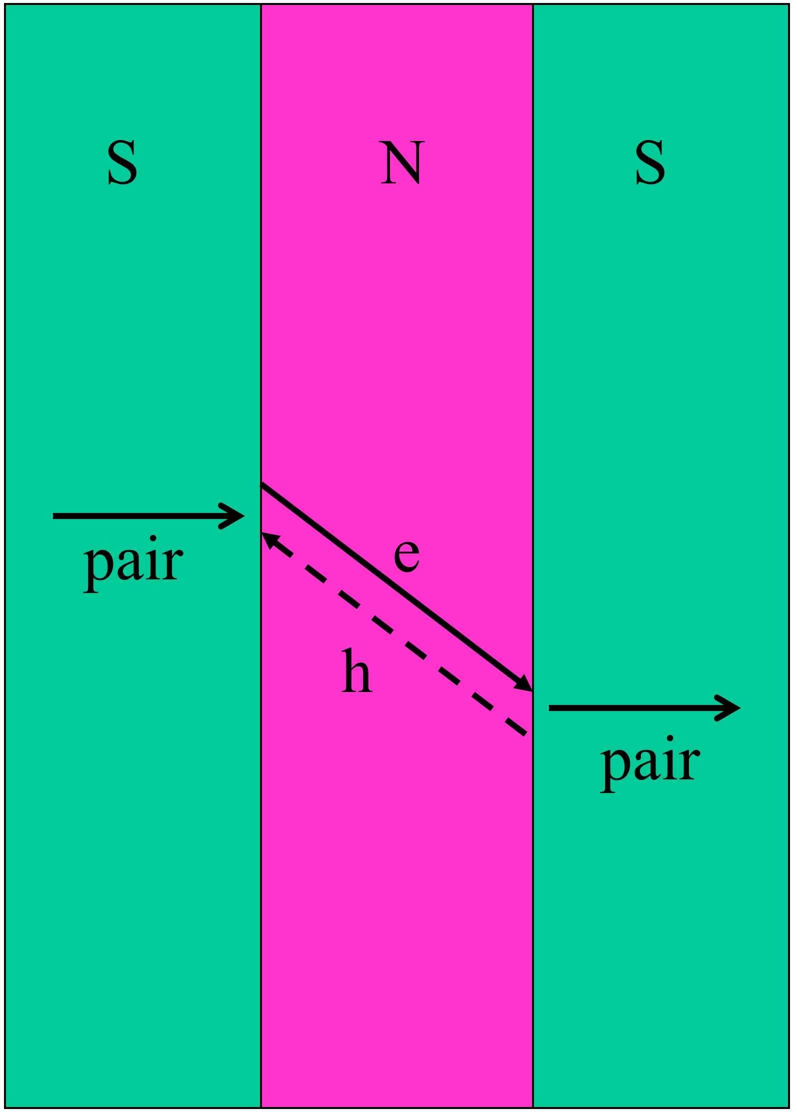




- A Superconductor-Normal metal-Superconductor sandwich where the weak link between superconductors occurs through the proximity effect.
- Andreev reflection at the N-S boundaries leads to sub-gap bound states that carry the pair current.
- Single-valuedness of the IV characteristic allows for non-latching technologies like RSFQ logic.
- Goal is to optimize the switching speed of these junctions as determined by IcRn

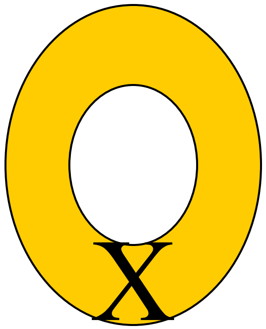
Andreev Bound States

- At an N-S interface an incoming electron from the normal metal can be reflected into a superconducting pair and a hole (especially at low energies).
- Reflection off both N-S boundaries leads to a bound state in the weak-link region.
- Since Andreev reflection is strongest for voltages less than the superconducting gap, most bound states are sub-gap states localized within the barrier.
- It is the left and right-moving "pieces" of these bound states that carry the Josephson current when there is a phase difference across the junction.

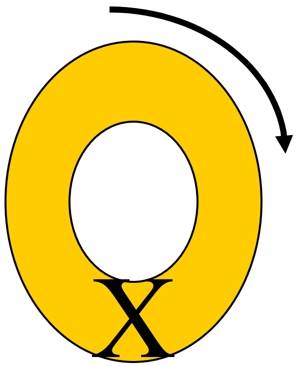


Digital Electronics and RSFQ logic

- Rapid single-flux quantum logic is used for high-speed applications. A loop of superconducting material has one JJ interrupting it. The absence or presence of a flux quantum in the loop is the binary 0 and 1 of the device. (Much faster than latching technologies.)
- The flux is changed by generating a voltage pulse through the junction, whose time integral is equal to a flux quantum. Since the voltage scale is set by the product IcRn, which is on the order of a few mV in low-Tc superconductors, operating speeds of up to 770 GHz have been already demonstrated.
- High Tc junctions, with IcRn products larger than 20 mV can possibly produce speeds in excess of 1 THz!
- The goal for fast electronics is to optimize the IcRn product of a junction.



Binary 0, no flux

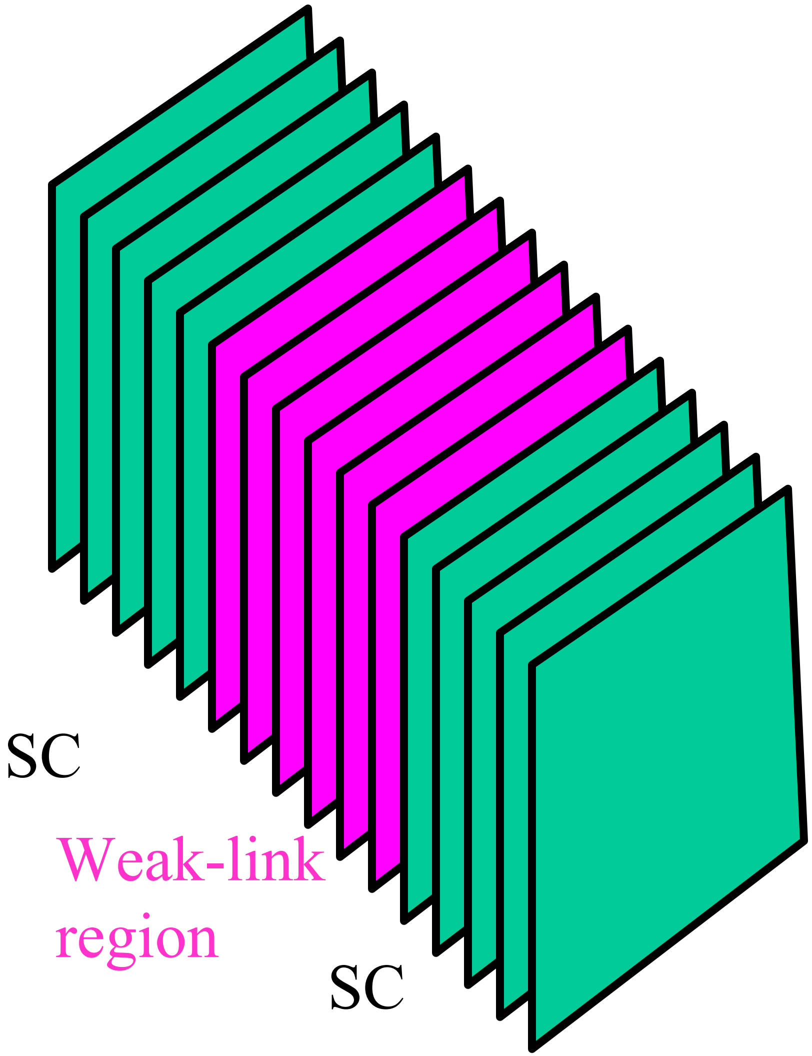


Binary 1, one flux quantum

Petaflop Computing

- Semiconductor chips run into overheating problems when run much faster than about 3GHz, where they will generate about 175 W of power.
- Conventional petaflop computers will require about a 10MW of power from 100,000 chips at 10 Gflops per chip.
- IBM's "blue-gene" project has a new architecture design using multithreads that can reduce the power to 1 MW. They will invest $100 million over the next five years to build it.
- Likharev's JJ petaflop computer requires 100 GHz CPU's made out of low-Tc junctions using sub-micron lithography.
- Power estimates run as low as 1-10KW, mainly being tied up in refrigeration down to 4K.
- Architecture design uses multithreading as well, and requires significant work in coordinating memory operations with CPU operations. It is estimated that $50 million could complete the project in five years, but it is not funded at that level.

Optimization of the speed of a JJ



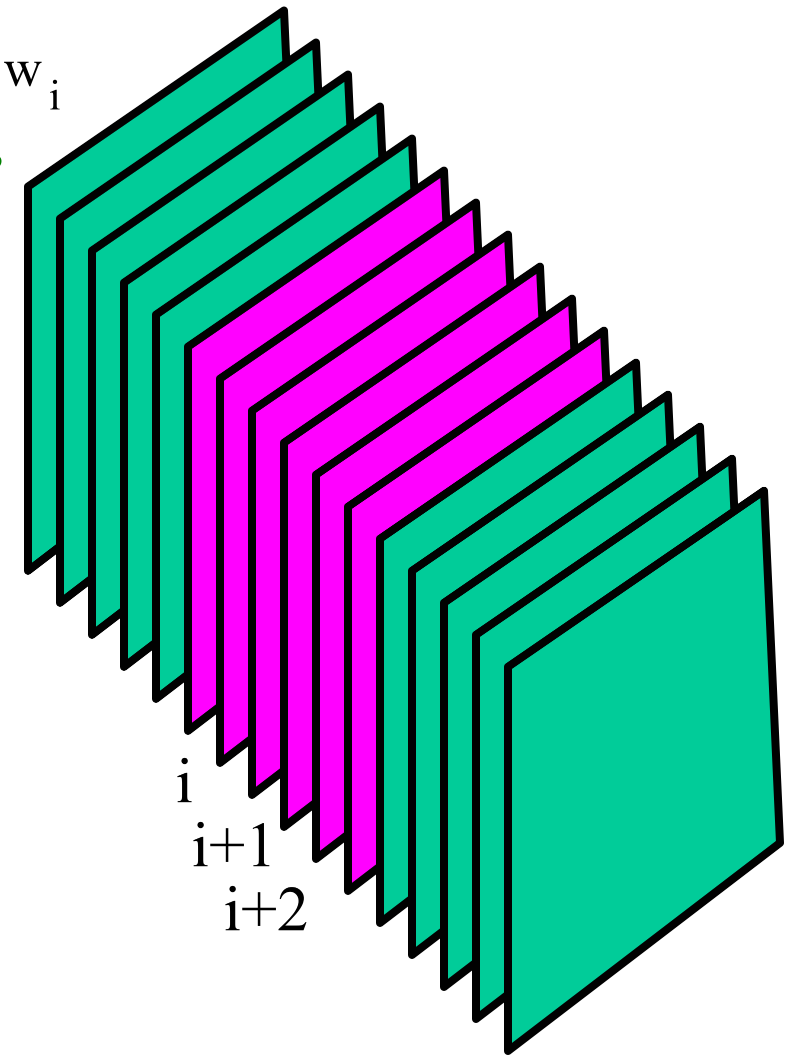
- Insulating barriers found in tunnel junctions have a high resistance and a low Josephson current.
- Metal barriers found in proximity-effect junctions have a low resistance and a high Josephson current.
- Is the speed optimized (i.e. the product IcRn maximized) when the barrier lies near the metal-insulator transition? What type of material produces the best Josephson junction weak-link region?
- Our plan is to adopt an efficient massively parallel materials-specific formalism to model and optimize the characteristics of a JJ.

Types of Metal-Insulator Transitions

- Anderson (disorder, mobility edge, dimensionality effects)
- Mott-Hubbard (correlations within a single band)
- Periodic Anderson Model (f-electrons, hybridization, and the Kondo effect)
- Holstein (bipolaron self-trapping)
- Falicov-Kimball (thermodynamics of localized and itinerant electrons) **This is the model considered here.**
- All require electron correlations to be described properly.

Many-Body Formalism

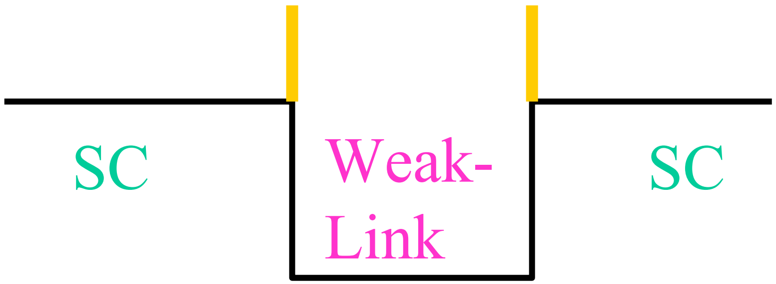
- Inhomogeneous system, with planes stacked along the z-direction.
- $H = -\Sigma\, t_{ij} c^*_{i\sigma} c_{j\sigma} + \Sigma\, U_i n_{i\uparrow} n_{i\downarrow} + \Sigma\, U^{FK}_i (n_{i\uparrow} + n_{i\downarrow})\, w_i$
- Hopping, site energy, Coulomb interaction, and the impurity interaction can vary from one plane to another.
- Local dynamical correlations are explicitly included for each plane via the dynamical mean field theory. The self-consistency relation is now modified to include effects that couple the effective medium between the planes.
- We illustrate the solution of the superconductor in the Hartree-Fock approximation, which is identical to a ***self-consistent solution*** of the Bogulubov-deGennes equations.

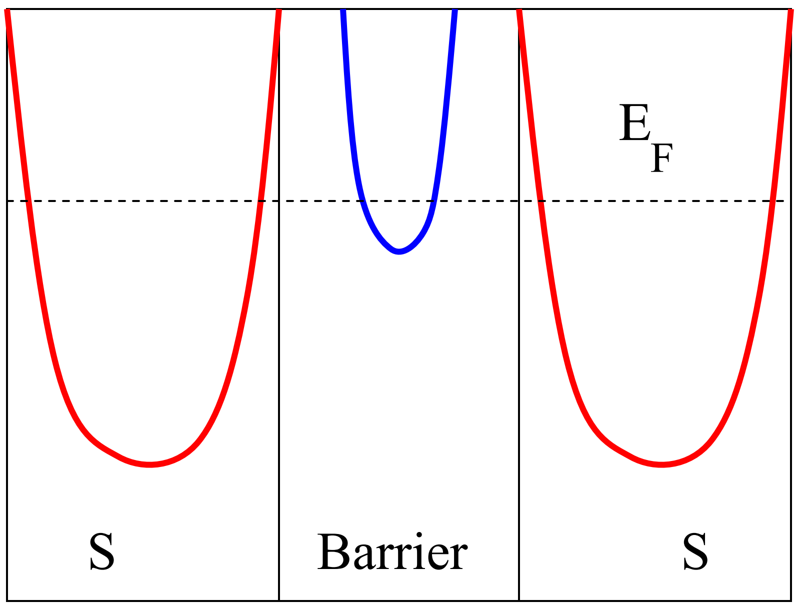


Continued-Fraction Technique

- We have devised an efficient technique to calculate the local Green's function at each site from the local self energy on each plane.
- Begin by performing a partial Fourier transform with respect to the transverse (in-plane) dimensions.
- Then use Dyson's equation to express the Green's functions in terms of the inverse of an infinite tridiagonal matrix which contains the information about the inhomogeneous spatial direction.
- A continued-fraction technique (similar to the Renormalized Perturbation Expansion) is then employed to ***exactly*** calculate the matrix elements of the inverse matrix, and hence the Green's functions.
- Finally, the results, which hold for each transverse momentum vector are summed to determine the local Green's function on each plane.
- This technique allows for a direct solution of the Bogoliubov-deGennes equations (and their generalizations) and is more than ***three orders of magnitude faster*** than direct application of the recursion method to the three-dimensional problem!

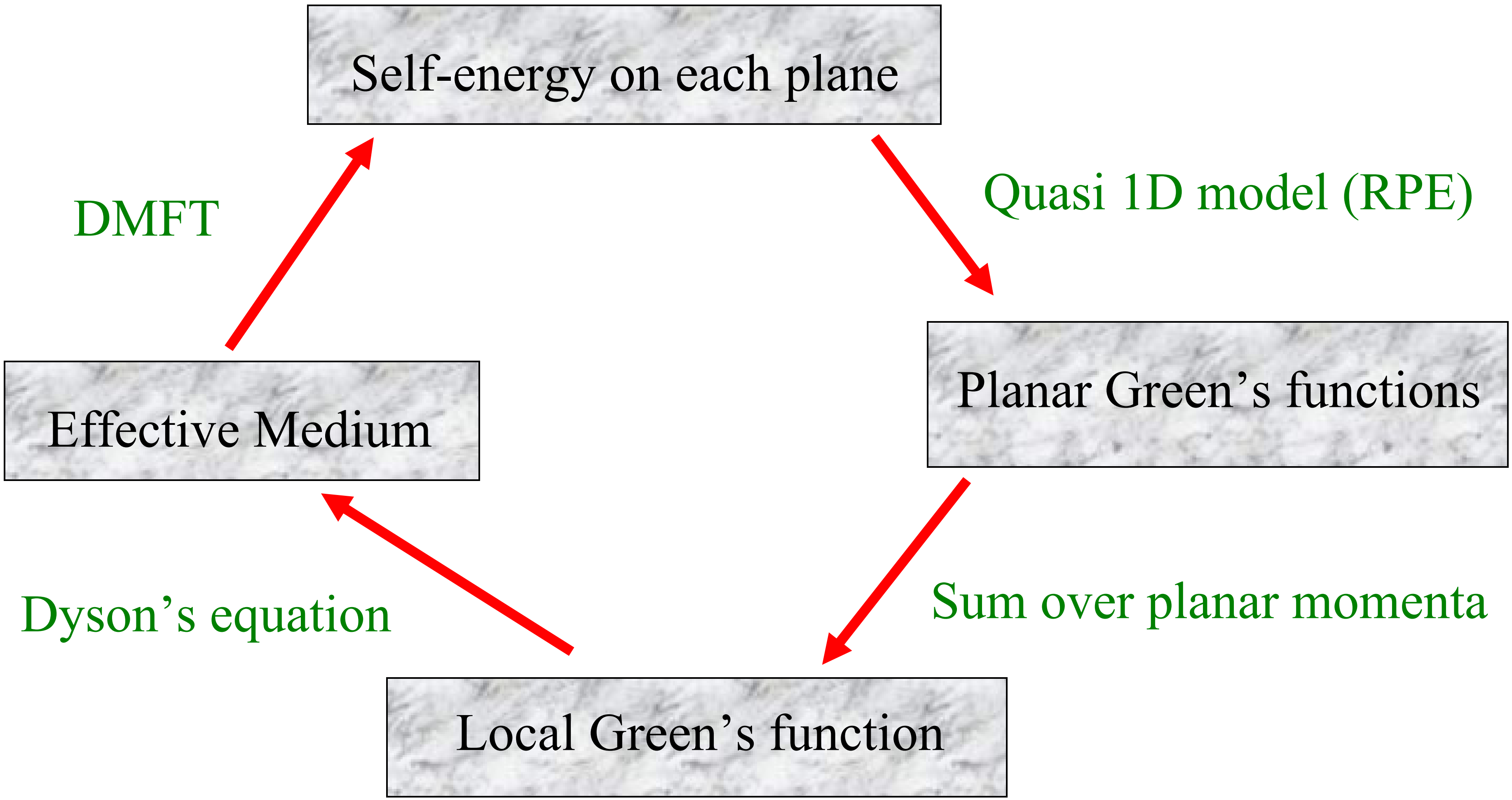
Conventional Models





- BTK model, interface scattering, no self-consistency, no electron-electron interactions in the barrier, no bandstructure effects. Simple exercise of matching boundary conditions for plane waves.
- Generalizations to include bandstructure effects (Fermi wavevector mismatch, varying effective mass) are easy to include.
- Self-consistency and especially correlations have been much more difficult.
- **All of these effects are automatically included in our approach!**

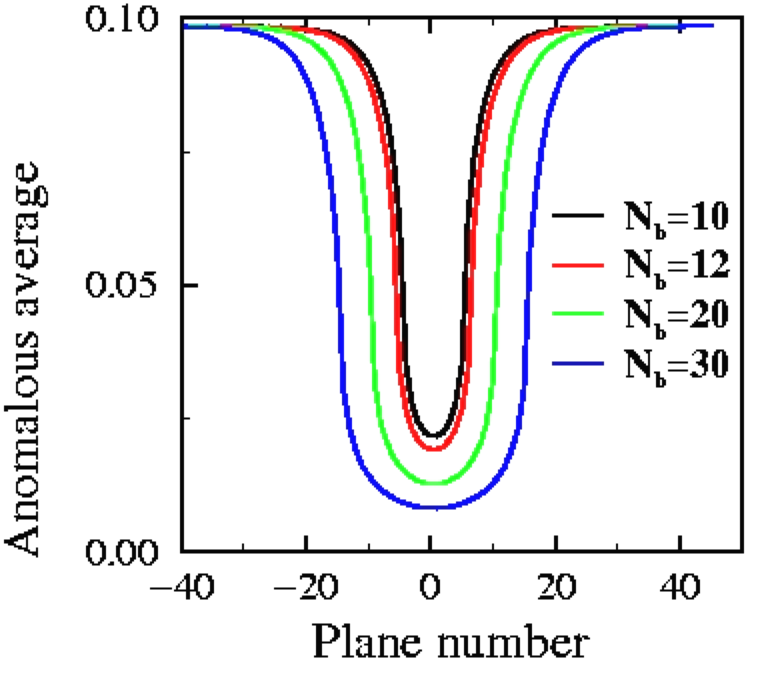
Self-Consistent loop of the Algorithm

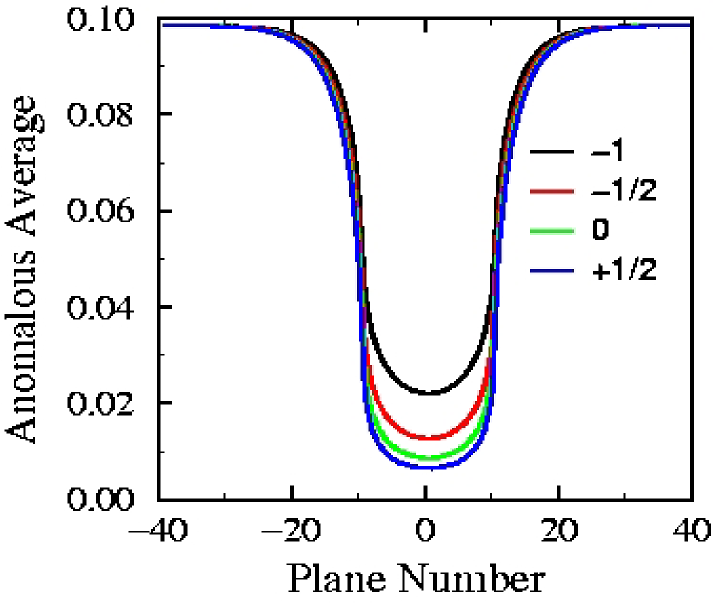


Algorithm is iterated until a self-consistent solution is achieved

Proximity Effect

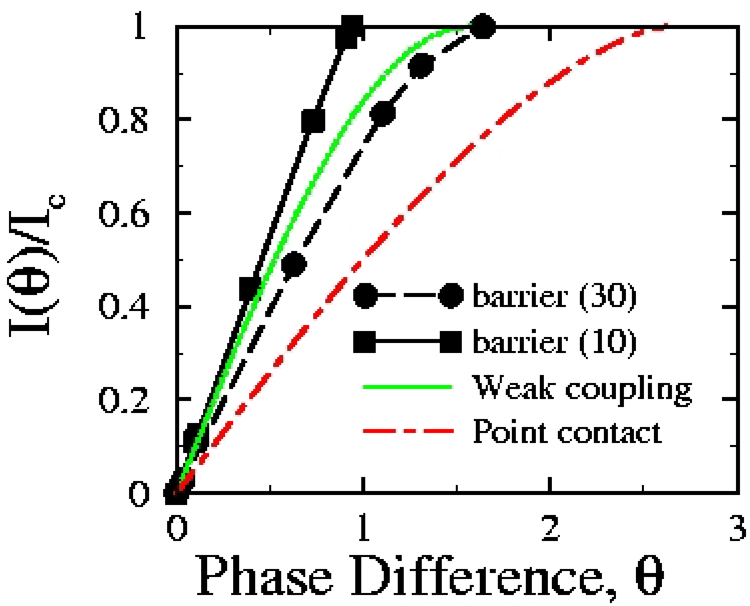
- Bulk superconductor has U= - 2 and half filling, which yields Tc=0.11, Δ = 0.198, and a coherence length of 10 lattice spacings.
- Note how the anomalous average decreases as the barrier width increases (the barrier has U=0).



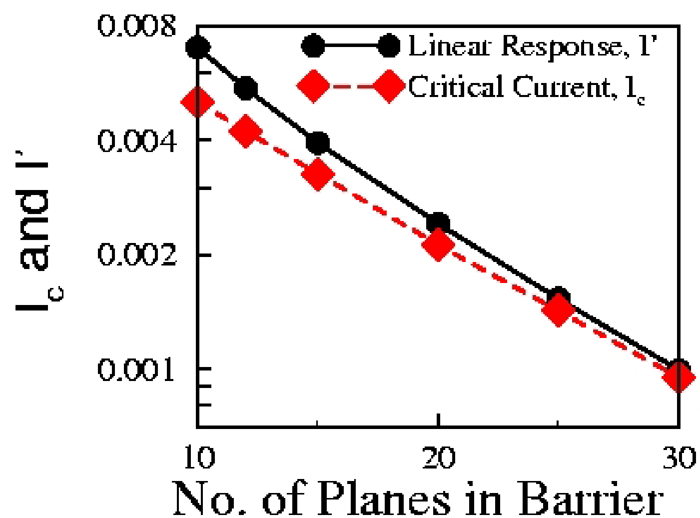


- Increasing the Coulomb interaction decreases the anomalous average, but the proximity effect survives even for repulsive values of the Coulomb interaction.

Current-phase relation



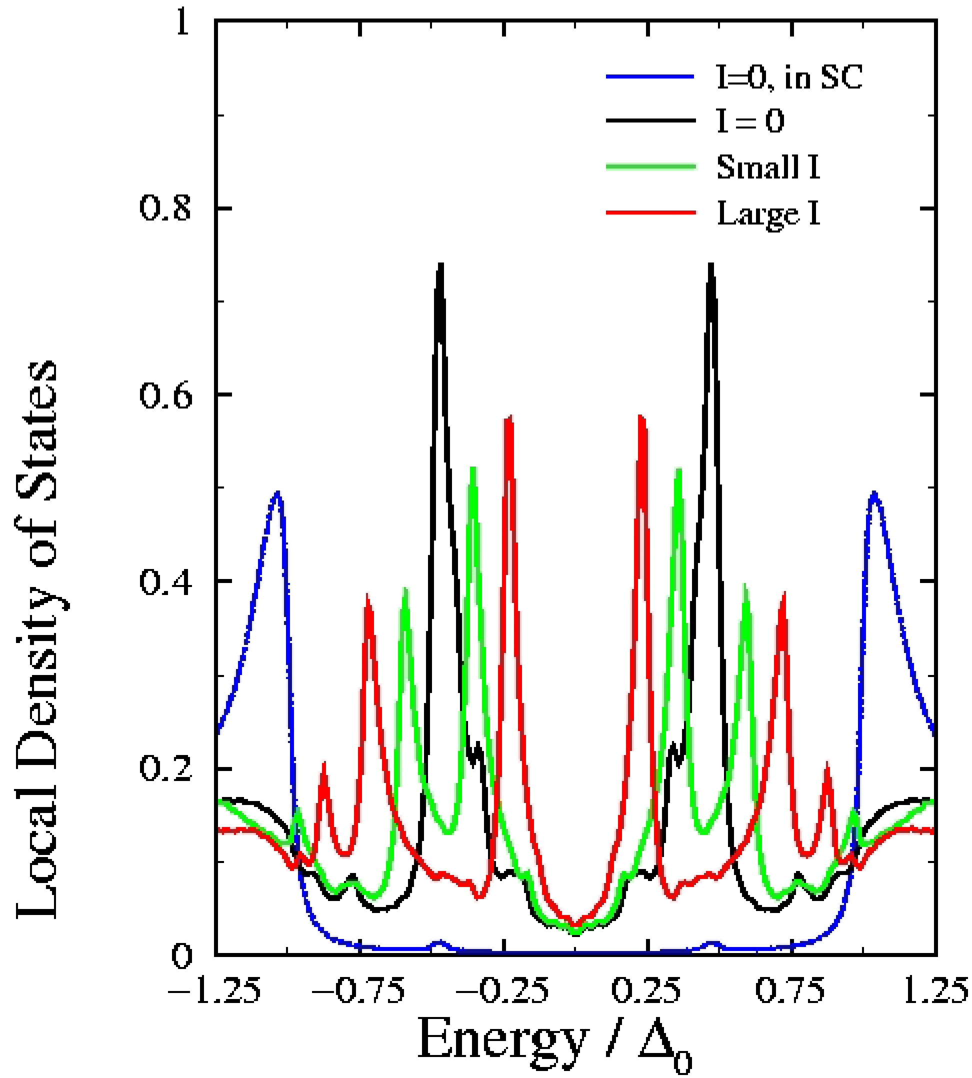
- Current-phase relation for a 10 and 30-site barrier versus the limiting forms I/Ic = sin θ and I/Ic=sin (θ/2).
- Note how the slope at θ=0 is nearly 1, indicating that I' is a good measure of Ic (and is an order of magnitude easier to calculate (because of the self-consistency)!



- Plot of the critical current Ic and the linear-response current I' show that they track well with each other over a wide range of barrier thicknesses.

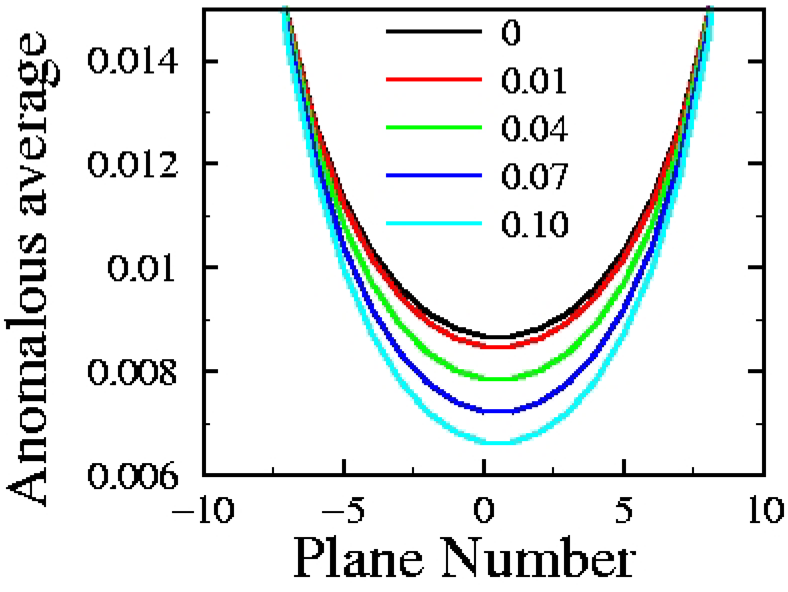
Many-Body Density of States

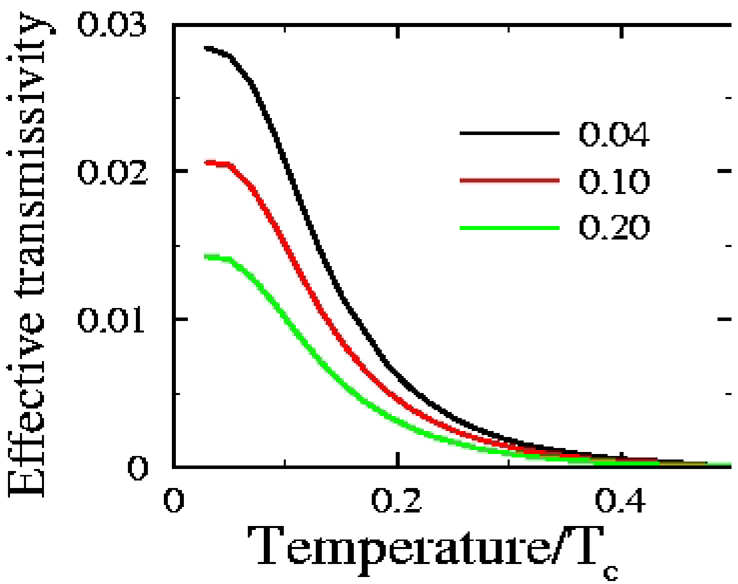
- The density of states is plotted at the center of the barrier for no current, small current, and large current. As the current increases, the splitting of the Andreev bound states into the left- and right-moving states becomes clear.
- For comparison, the density of states is shown just inside the superconductor, where the effects of the Andreev bound states can be seen arising from the self-consistency.
- This is a theoretical spectroscopy showing how barrier states rearrange as current flows.



Effect of charged impurities

- Adding charged impurities to the barrier creates scattering that reduces the anomalous average in apparent “violation” of Anderson’s theorem. Of course, the proximity effect is modified by charged scatterers.

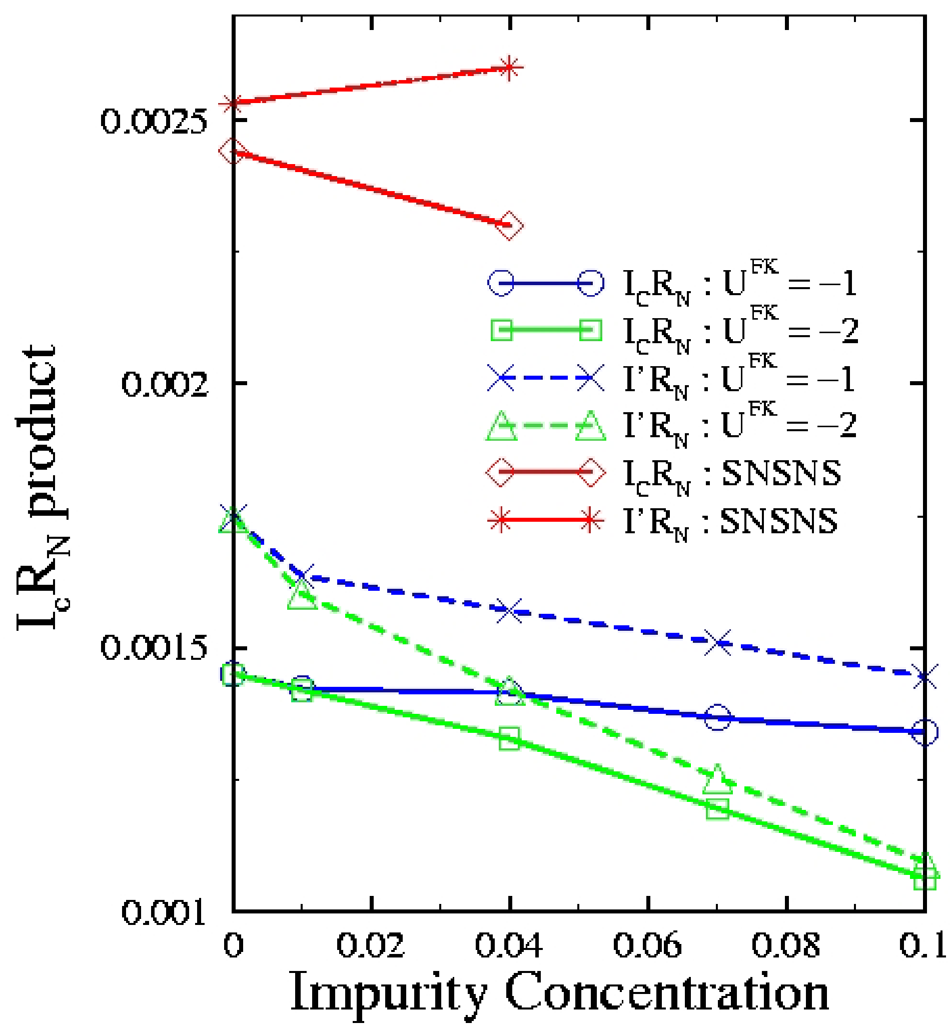




- The effective transmissivity of the barrier can be extracted from the analytic BCS-like formulae. We find that self-consistency leads to a strongly temperature dependent transmissivity for low transparency junctions.

Optimization of junction speed

- The dc-conductivity can be calculated with a Kubo formula neglecting vertex corrections.
- We find that in general, the IcRn product is maximized for junctions in the clean limit (with no impurity scattering).
- However, the IcRn product of SNSNS junctions which have a thin superconducting layer included in the barrier is enhanced. The additional scattering of the new interface and the enhancement of the superconductivity combine to increase both Ic and Rn. I'Rn even increases with charge scattering! Similar effects seen by Ketterson.



Conclusions

- Presented a new formalism that provides an efficient means to self-consistently determine the properties of Josephson junctions from a microscopic model. This allows both correlations and retardation effects to be included in the description of a junction.
- Future work will include an examination of materials-specific modeling such as Nb-InAs-Nb, Nb-Nb-doped SrTiO3-Nb, and conventional Nb-Al-AlO-Nb junctions.
- Calculations are possible for d-wave superconductors employing either the local approximation, or more sophisticated techniques such as the dynamical cluster approximation.
- Other effects such as charge redistribution due to work function mismatch or Schottky barriers as well as nonequilibrium I-V characteristics and subgap structures can also be examined.