

Ultrafast Digital Electronics: Optimizing the speed of a Josephson junction



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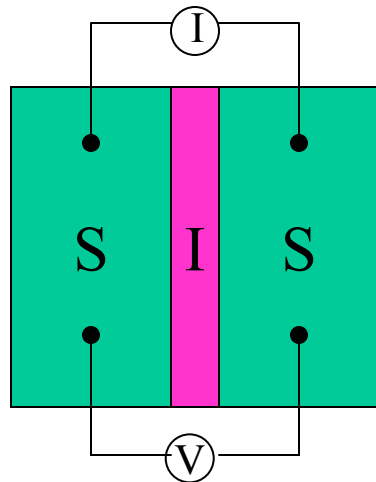
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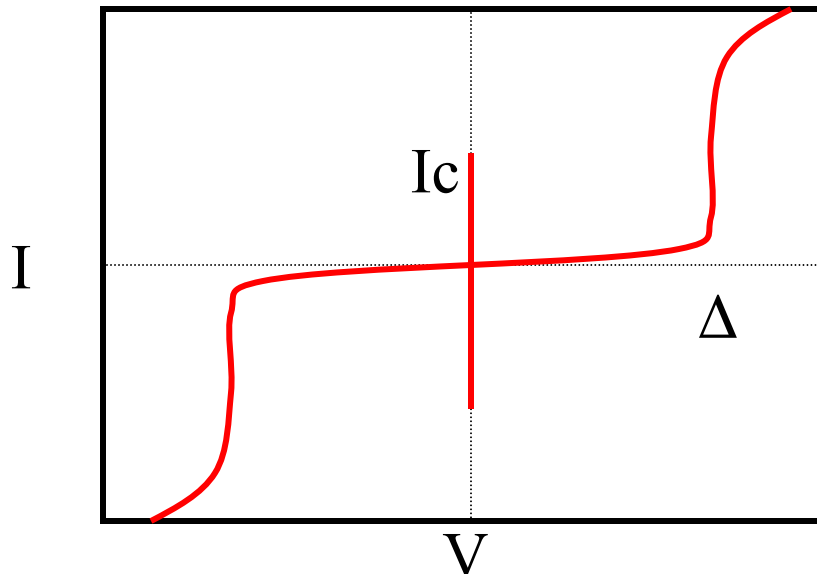
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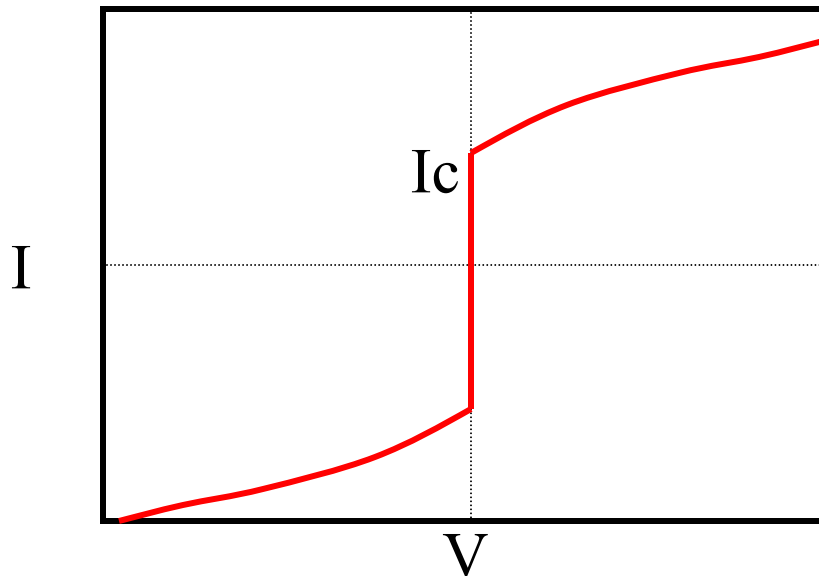
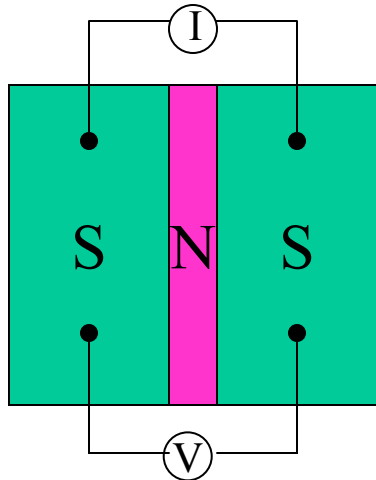
Josephson Tunnel Junctions



- A Superconductor-Insulator-Superconductor sandwich can tunnel coherent Cooper pairs (Josephson current) or can tunnel broken pairs (quasiparticles) through the barrier.
- If the phases of the superconducting wavefunctions differ, then there is a DC Josephson current $I = I_c \sin \theta$.
- The I-V characteristic is highly nonlinear at low voltages, leading to the possibility of important electronics applications (based on latching technologies which are slow and subject to “punch-through”).



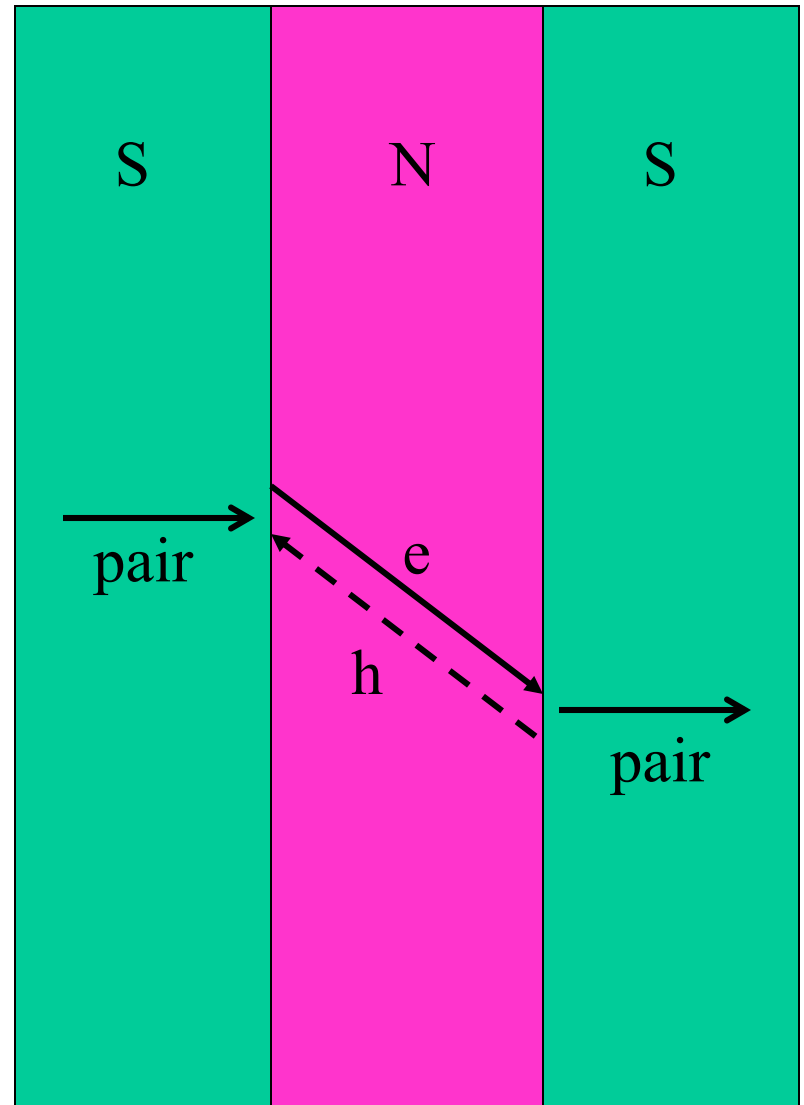
Josephson Proximity-Effect Junctions



- A Superconductor-Normal metal-Superconductor sandwich where the weak link between superconductors occurs through the proximity effect.
- Andreev reflection at the N-S boundaries leads to sub-gap bound states that carry the pair current.
- Single-valuedness of the IV characteristic allows for non-latching technologies like RSFQ logic.
- Goal is to optimize the switching speed of these junctions as determined by $I_c R_n$.

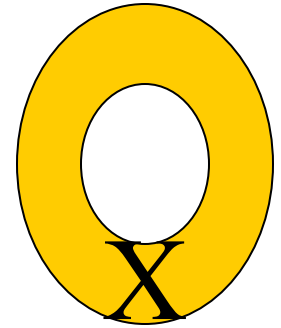
Andreev Bound States

- At an N-S interface an incoming electron from the normal metal can be reflected into a superconducting pair and a hole (especially at low energies).
- Reflection off both N-S boundaries leads to a bound state in the weak-link region.
- Since Andreev reflection is strongest for voltages less than the superconducting gap, most bound states are sub-gap states localized within the barrier.
- It is the left and right-moving “pieces” of these bound states that carry the Josephson current when there is a phase difference across the junction.

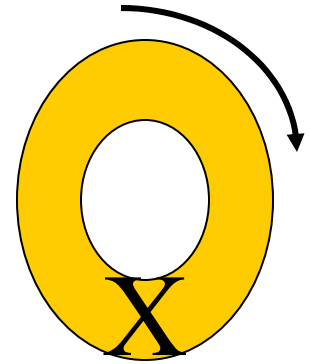


Digital Electronics and RSFQ logic

- Rapid single-flux quantum logic is used for high-speed applications. A loop of superconducting material has one JJ interrupting it. The absence or presence of a flux quantum in the loop is the binary 0 and 1 of the device. (Much faster than latching technologies.)
- The flux is changed by generating a voltage pulse through the junction, whose time integral is equal to a flux quantum. Since the voltage scale is set by the product $I_c R_n$, which is on the order of a few mV in low- T_c superconductors, operating speeds of up to 770 GHz have been already demonstrated.
- High T_c junctions, with $I_c R_n$ products larger than 20 mV can possibly produce speeds in excess of 1 THz!
- The goal for fast electronics is to optimize the $I_c R_n$ product of a junction.



Binary 0, no flux



Binary 1, one flux quantum

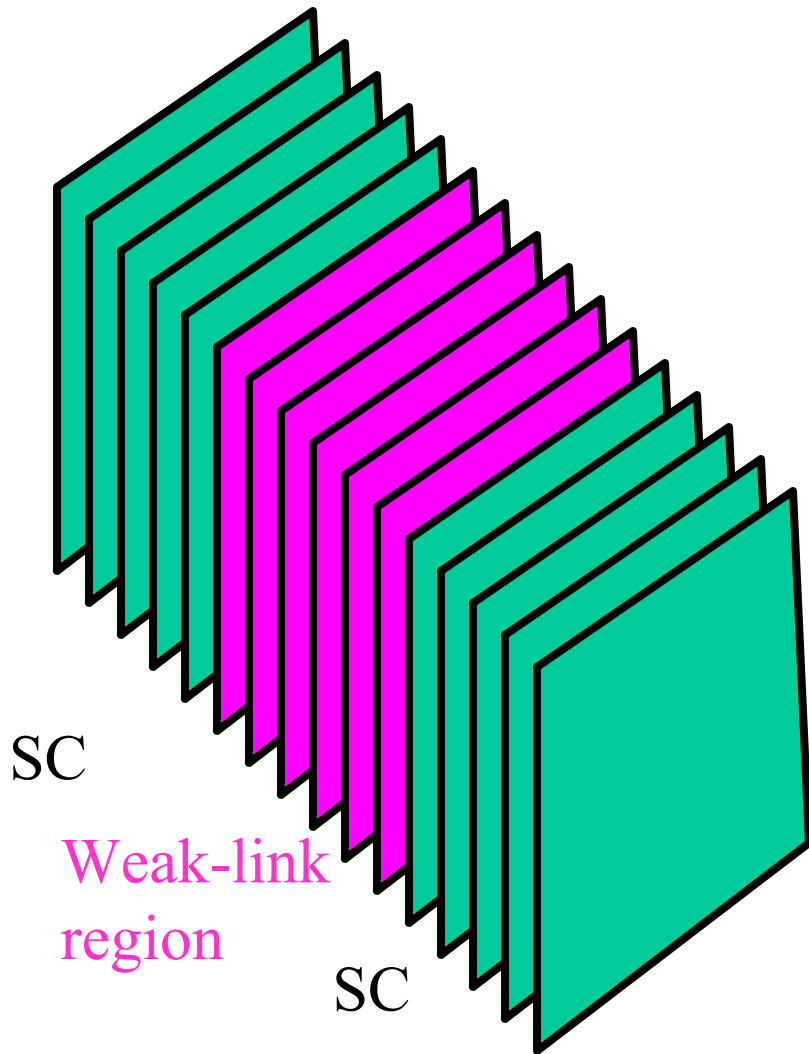
Navy Interest

- High precision, high speed, analog-digital converters for the Advanced Multifunction Radio Frequency System for use in radar, electronic warfare, and communications.
- Long-term specs include 12 bit ADC with 1GHz of bandwidth. **Speed is more important than resolution.**
- Short-term goal is a 20 bit ADC with 20MHz of bandwidth.
- Employ High-Temperature Superconductor Technology.
- The theoretical calculation and model is a scalable massively parallel solution to a scientific problem.
- Work could also have an impact on a JJ-based petaflop computer which could be competitive with IBM's blue-gene project.

Petaflop Computing

- Semiconductor chips run into overheating problems when run much faster than about 3GHz, where they will generate about 175 W of power.
- Conventional petaflop computers will require about a 10MW of power from 100,000 chips at 10 Gflops per chip.
- IBM's "blue-gene" project has a new architecture design using multithreads that can reduce the power to 1 MW. They will invest \$100 million over the next five years to build it.
- Likharev's JJ petaflop computer requires 100 GHz CPU's made out of low-Tc junctions using sub-micron lithography.
- Power estimates run as low as 1KW, mainly being tied up in refrigeration down to 4K.
- Architecture design uses multithreading as well, and requires significant work in coordinating memory operations with CPU operations. It is estimated that \$50 million could complete the project in five years, but it is not funded at that level.

Optimization of the speed of a JJ



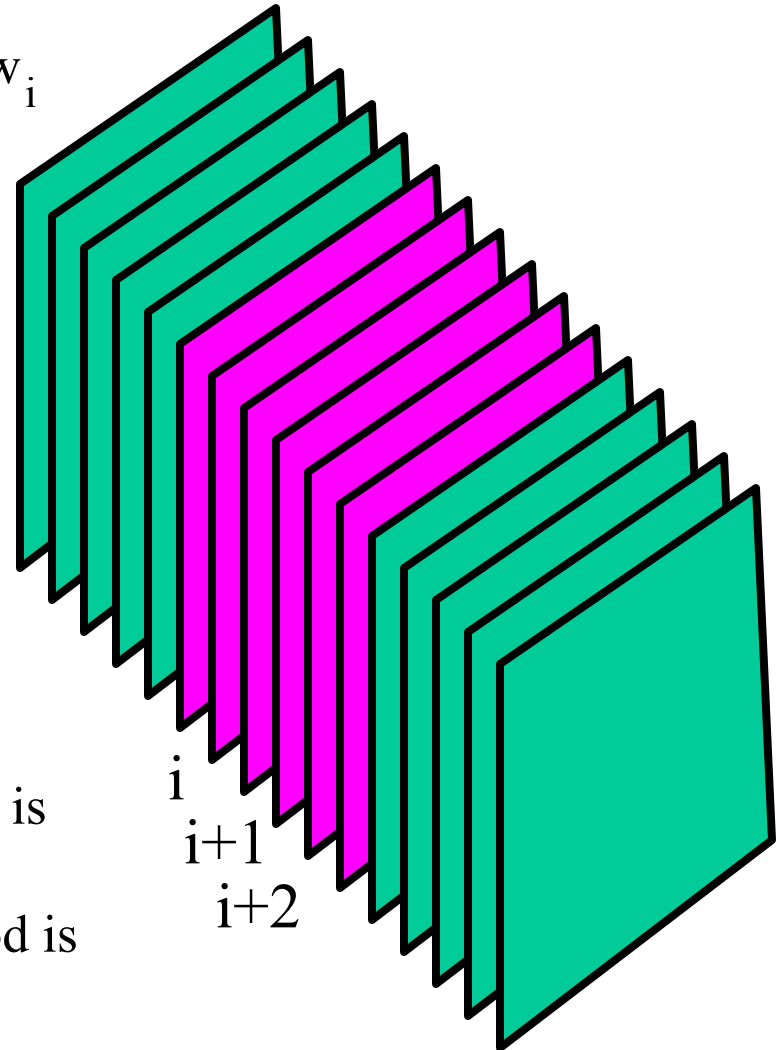
- Insulating barriers found in tunnel junctions have a high resistance and a low Josephson current.
- Metal barriers found in proximity-effect junctions have a low resistance and a high Josephson current.
- Is the speed optimized (i.e. the product $I_c R_n$ maximized) when the barrier lies near the metal-insulator transition? What type of material produces the best Josephson junction weak-link region?
- Our plan is to adopt an efficient massively parallel materials-specific formalism to model and optimize the characteristics of a JJ.

Types of Metal-Insulator Transitions

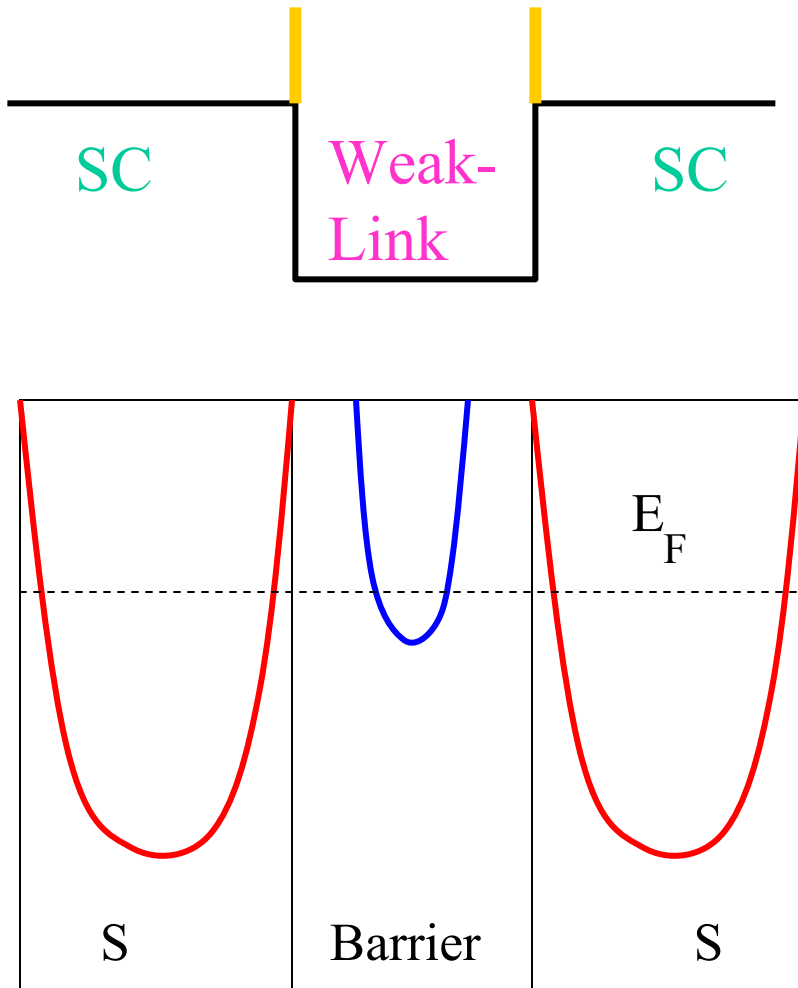
- Anderson (disorder, mobility edge, dimensionality effects)
- Mott-Hubbard (correlations within a single band)
- Periodic Anderson Model (f-electrons, hybridization, and the Kondo effect)
- Holstein (bipolaron self-trapping)
- Falicov-Kimball (thermodynamics of localized and itinerant electrons) **This is the model considered here.**
- All require electron correlations to be described properly.

Many-Body Formalism

- Inhomogeneous system, with planes stacked along the z-direction.
- $H = -\sum_{ij} t_{ij} c_{i\sigma}^* c_{j\sigma} + \sum_i U_i n_{i\uparrow} n_{i\downarrow} + \sum_i^{FK} U_i (n_{i\uparrow} + n_{i\downarrow}) w_i$
- Hopping, site energy, Coulomb interaction, and the impurity interaction can vary from one plane to another.
- Local dynamical correlations are explicitly included for each plane via the dynamical mean field theory. The self-consistency relation is now modified to include effects that couple the effective medium between the planes.
- We illustrate the solution of the superconductor in the H-F approximation, which is identical to a *self-consistent solution* of the Bogulubov-deGennes equations (our method is 1000 times faster than conventional ones).



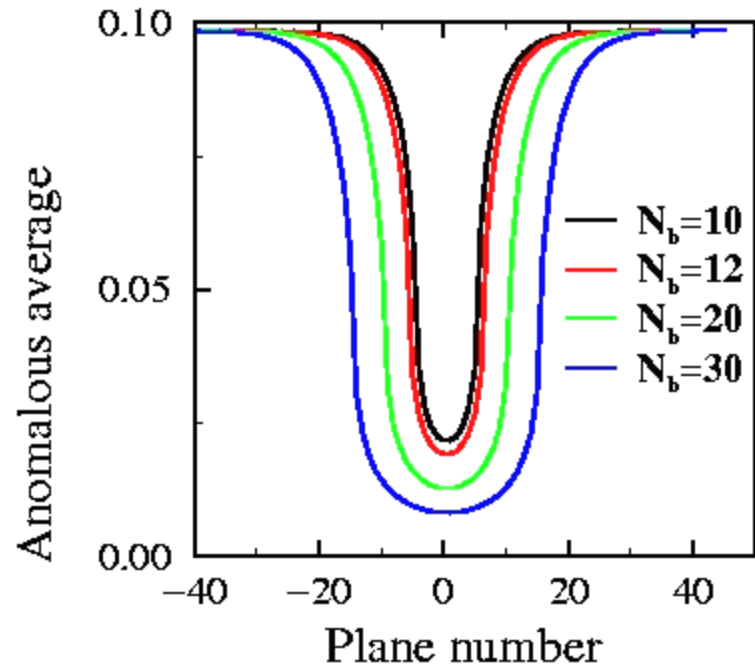
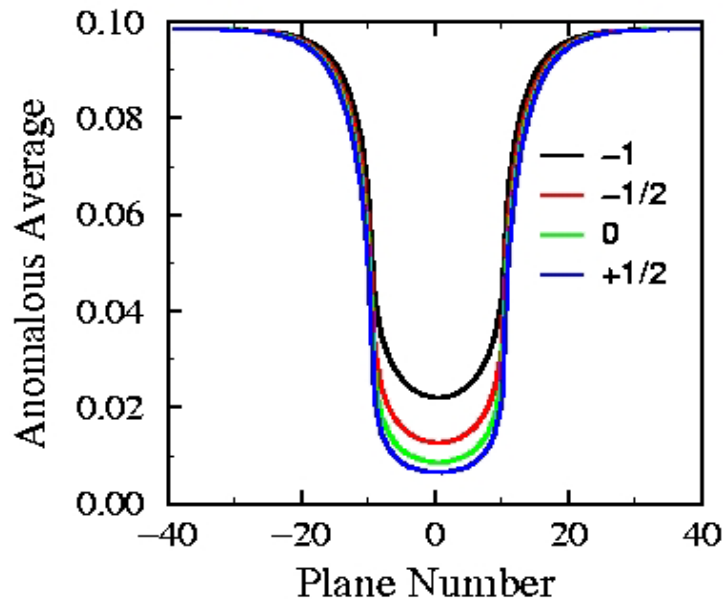
Conventional Models



- BTK model, interface scattering, no self-consistency, no electron-electron interactions in the barrier, no bandstructure effects. Simple exercise of matching boundary conditions for plane waves.
- Generalizations to include bandstructure effects (Fermi wavevector mismatch, varying effective mass) are easy to include.
- Self-consistency and especially correlations have been much more difficult.
- **All of these effects are automatically included in our approach!**

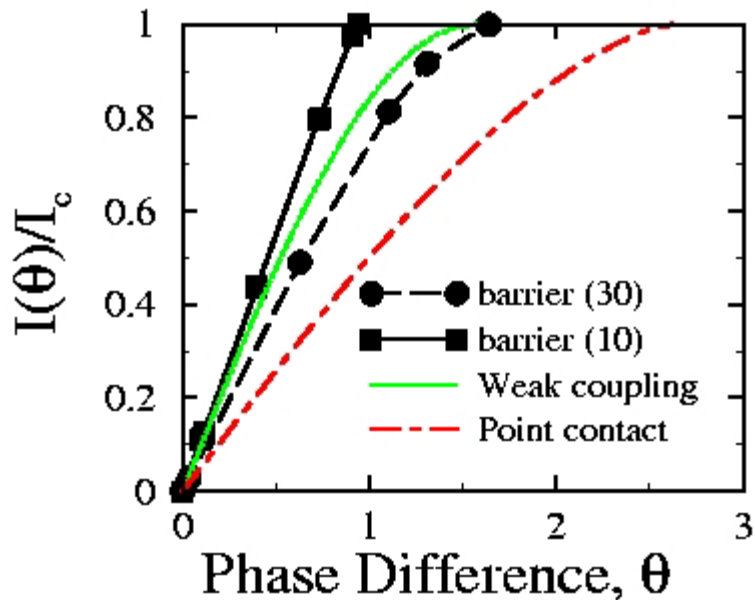
Proximity Effect

- Bulk superconductor has $U = -2$ and half filling, which yields $T_c = 0.11$, $\Delta = 0.198$, and a coherence length of 10 lattice spacings.
- Note how the anomalous average decreases as the barrier width increases (the barrier has $U = 0$).



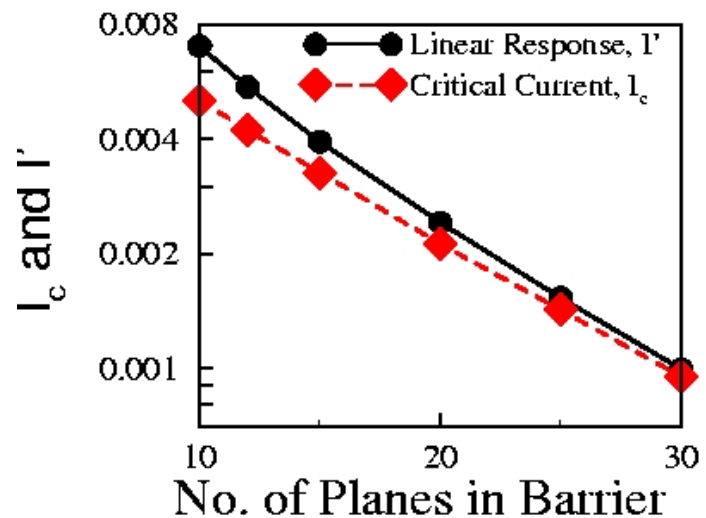
- Increasing the Coulomb interaction decreases the anomalous average, but the proximity effect survives even for repulsive values of the Coulomb interaction.

Current-phase relation



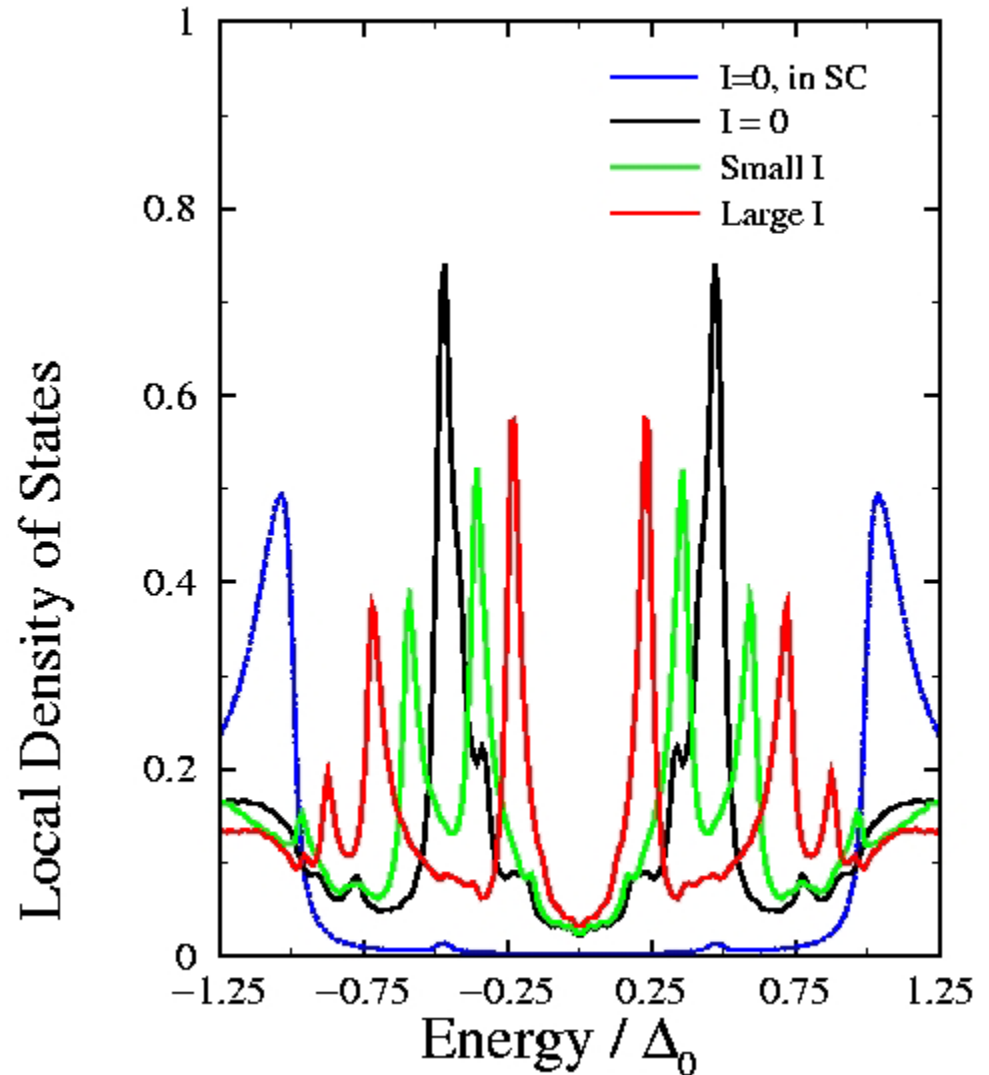
- Plot of the critical current I_c and the linear-response current I' show that they track well with each other over a wide range of barrier thicknesses.

- Current-phase relation for a 10 and 30-site barrier versus the limiting forms $I/I_c = \sin \theta$ and $I/I_c = \sin(\theta/2)$.
- Note how the slope at $\theta=0$ is nearly 1, indicating that I' is a good measure of I_c (and is an order of magnitude easier to calculate because of the self-consistency)!



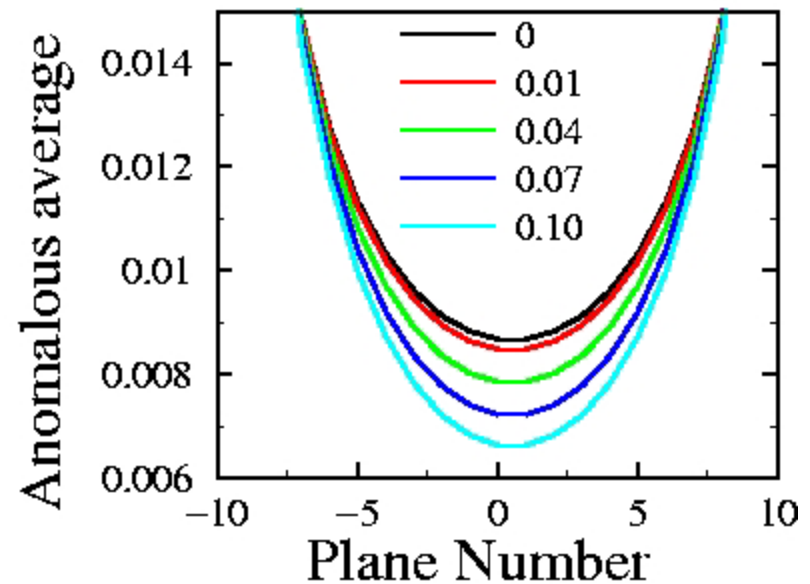
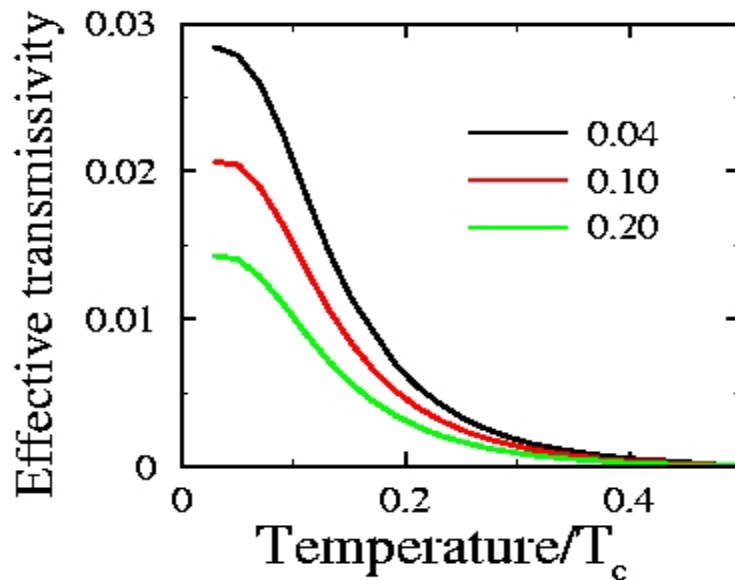
Many-Body Density of States

- The density of states is plotted at the center of the barrier for no current, small current, and large current. As the current increases, the splitting of the Andreev bound states into the left- and right-moving states becomes clear.
- For comparison, the density of states is shown just inside the superconductor, where the effects of the Andreev bound states can be seen arising from the self-consistency.



Effect of charged impurities

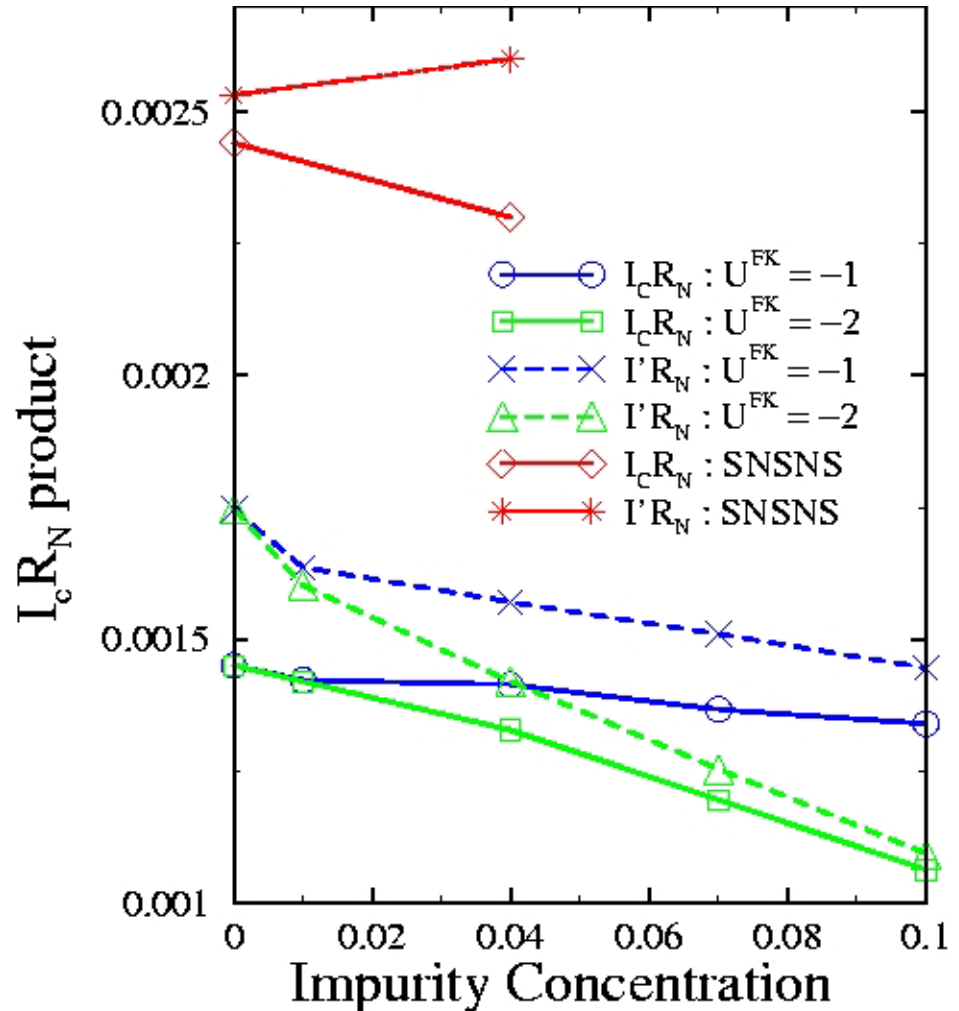
- Adding charged impurities to the barrier creates scattering that reduces the anomalous average in apparent “violation” of Anderson’s theorem. Of course, the proximity effect is modified by charged scatterers.



- The effective transmissivity of the barrier can be extracted from the analytic BCS-like formulae. We find that self-consistency leads to a strongly temperature dependent transmissivity for low transparency junctions.

Optimization of junction speed

- The dc-conductivity can be calculated with a Kubo formula neglecting vertex corrections.
- We find that in general, the $I_c R_n$ product is maximized for junctions in the clean limit (with no impurity scattering).
- However, the $I_c R_n$ product of SNSNS junctions which have a thin superconducting layer included in the barrier is enhanced. The additional scattering of the new interface and the enhancement of the superconductivity combine to increase both I_c and R_n . $I' R_n$ even increases with charge scattering!



Outstanding Technical Issues

- The formalism has reached a stage where materials-specific modeling of low- T_c systems is now possible. Effects due to charge redistribution (Schottky barriers or accumulation layers) can be treated with a long-range Coulomb interaction and materials specific electronic bandstructure and electron-phonon spectral functions can be self-consistently included. What is needed is data on $I_c(T)$ and R_n for well characterized experimental systems to benchmark the theory. We have two collaborations to work in these areas:
 - (Price and Rogers) Nb superconductor and Nb-doped STO weak-link; YBCO superconductor and Nb-doped STO weak-link.
 - (Greene) Nb superconductor and InAs weak-link.
- Two remaining issues for the theory are to generalize the formalism to include d-wave superconductors and to include nonequilibrium effects to generate IV characteristics.

Conclusions

- Presented a new formalism that provides an efficient means to self-consistently determine the properties of Josephson junctions from a microscopic model. This allows both correlations and retardation effects to be included in the description of a junction.
- Future work will include an examination of materials-specific modeling such as Nb-InAs-Nb or Nb-Nb-doped SrTiO₃-Nb junctions. We will examine Schottky barriers and charge accumulation layers in addition to including the correct electron-phonon spectral functions and electronic density of states in the modeling.
- Calculations are possible for d-wave superconductors employing either the local approximation, or more sophisticated techniques such as the dynamical cluster approximation.
- Non-equilibrium effects can also be modeled in a Keldysh formalism. This will allow one to also determine I-V characteristics.