Building an optical lattice emulator: Theory and experiment

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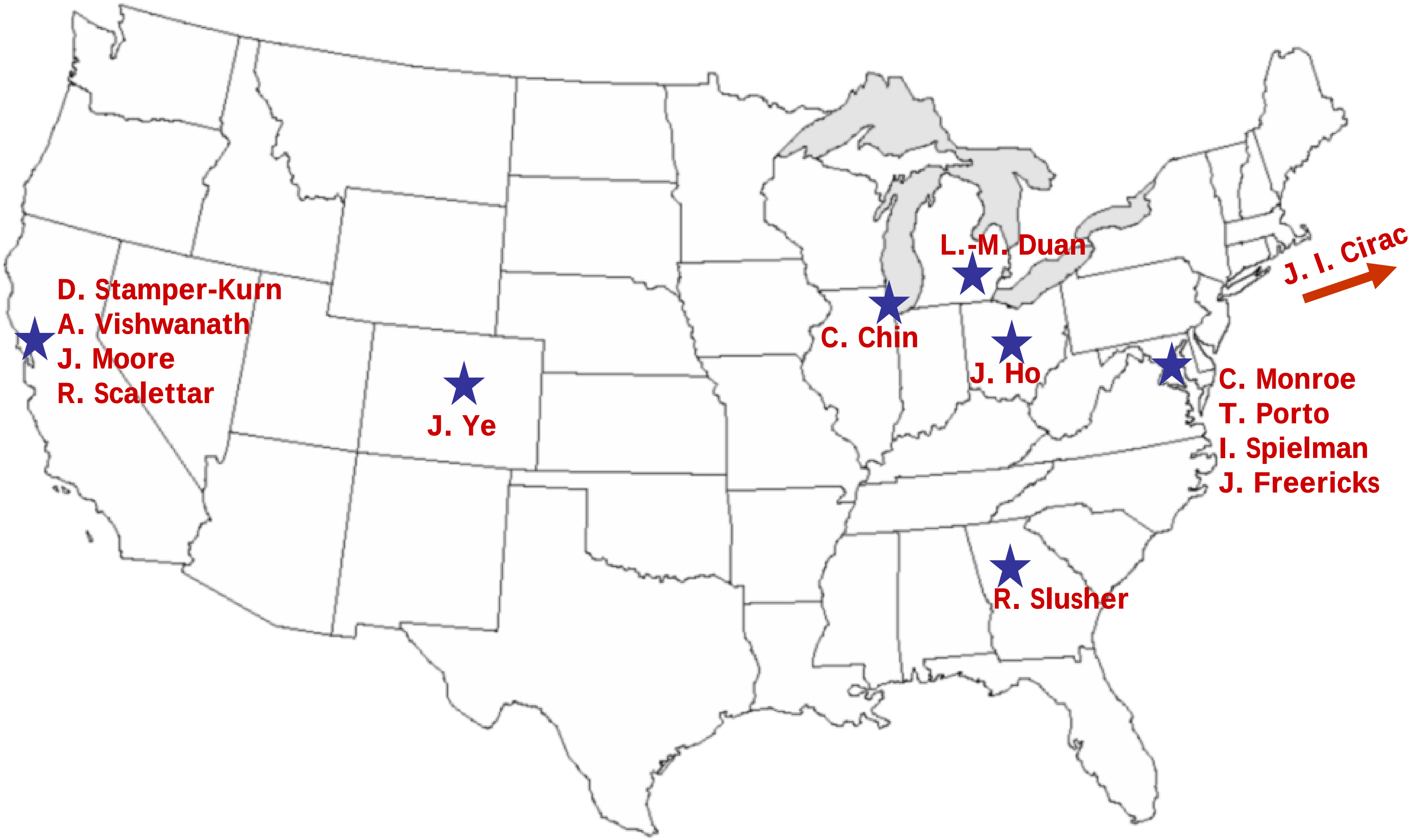
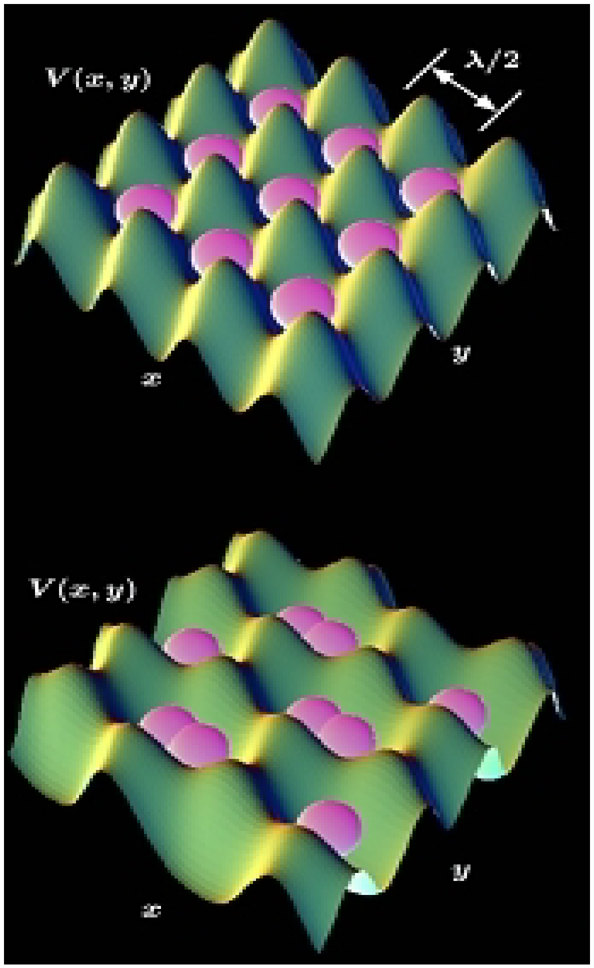
Location of team members



Simulations of quantum magnetism in two complentary AMO platforms:

Ion Traps

Optical lattices

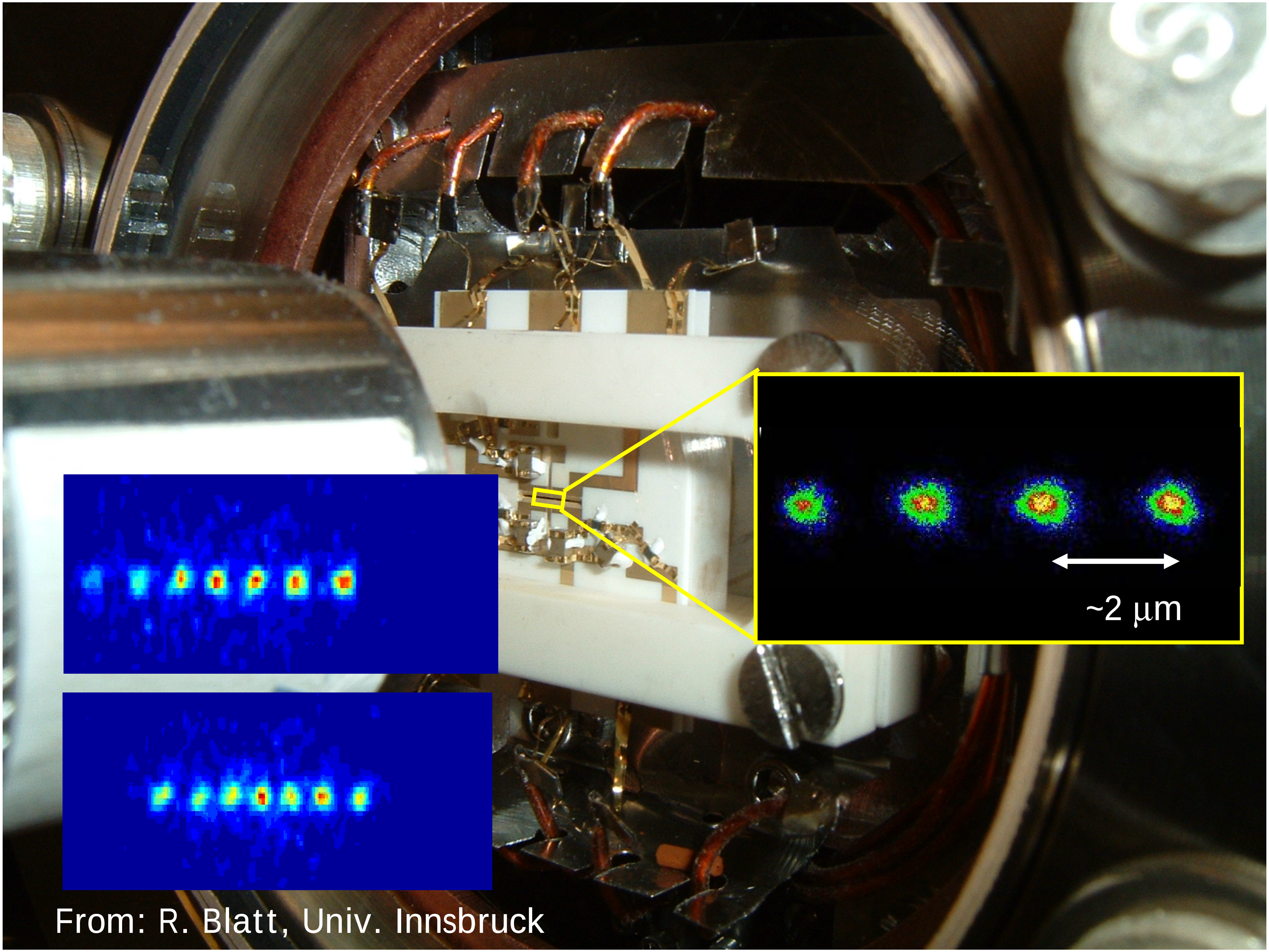


Role of theory in the OLE project

(1) Accurate calculation of phase diagrams for benchmarking

(2) Postprocessing of OLE data to find homogeneous (bulk) phase diagrams

Ion trap OLE in a linear chain


~2 µm
From: R. Blatt, Univ. Innsbruck

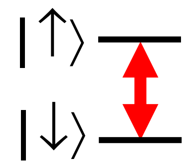
Symmetries of the Hamiltonian

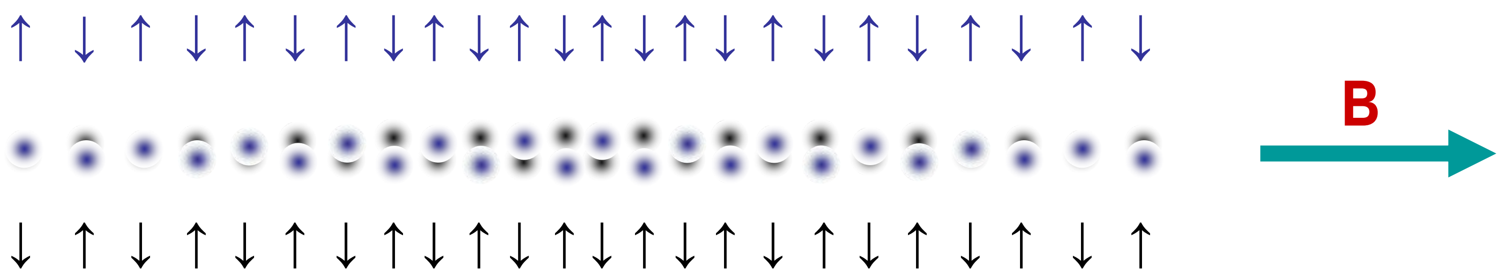
$$H = \sum_{ij} (J_{ij}^x S_i^x S_j^x + J_{ij}^y S_i^y S_j^y + J_{ij}^z S_i^z S_j^z) - \sum_i h_i \bullet S_i$$

- General XYZ spin model in an external field
- The XXZ model with a field in the z-direction has S^z as a good quantum number
- The isotropic model has S^2 as a good quantum number.

Creating an effective B field

ADD: Independent spin flips
(e.g., microwaves)
effective axial B-field





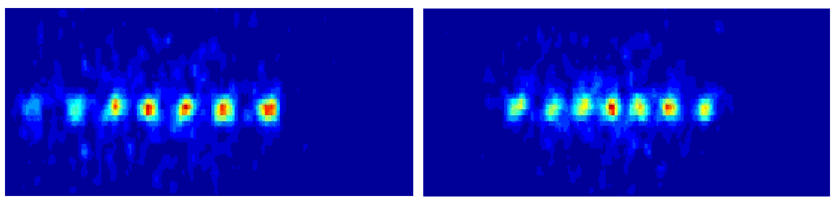
GROUND STATE: Antiferromagnetic ordering

Controlling the exchange $J_{i,j}$

$J_{i,j}$ proportional to:

- laser intensity at ion i
- laser intensity at ion j
- contribution of ion i to relevant motional mode
- contribution of ion j to relevant motional mode

axial
(ferrom.)



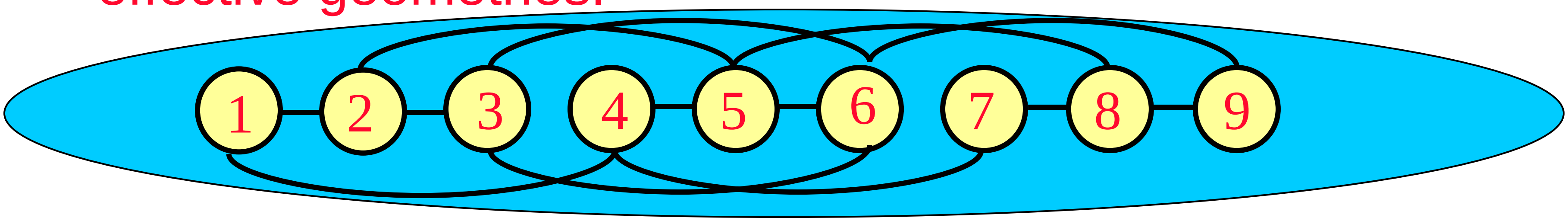
transverse
(antiferrom.)

zig-zag

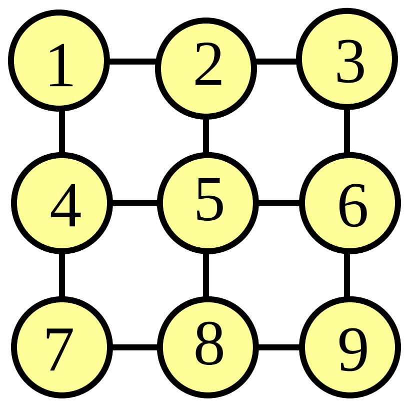
Operations with transverse modes: Zhu, et al., PRL 97, 050505 (2006)

Stroboscopic Hamiltonian

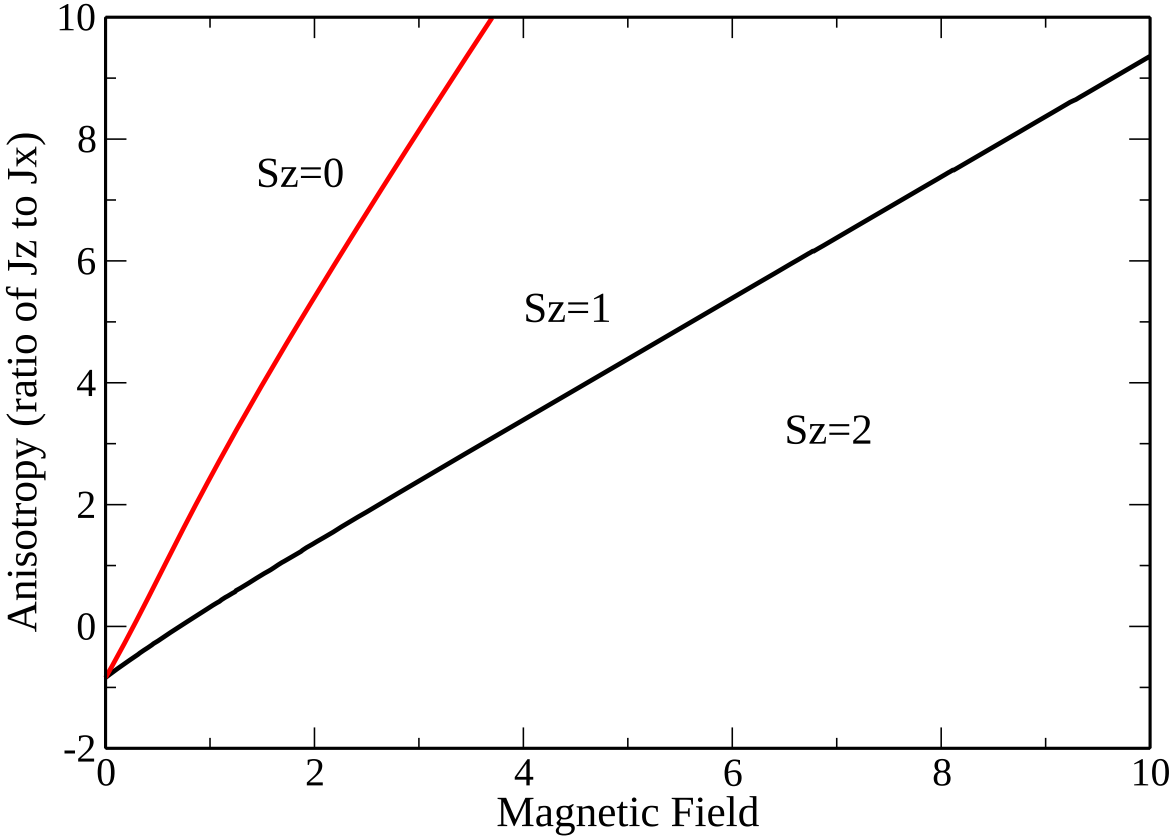
- **Effective (time-averaged) Heisenberg spin model.** Couplings often range beyond nearest neighbor and are inhomogeneous due to irregular spacing of ions. Although ions are arranged geometrically in a chain, varying couplings between different sites produces different effective geometries.

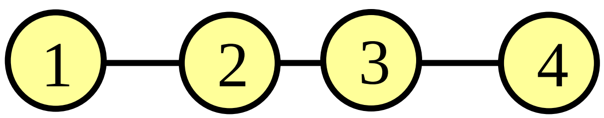


Mapping of an nine site chain onto a square lattice.



XZ model





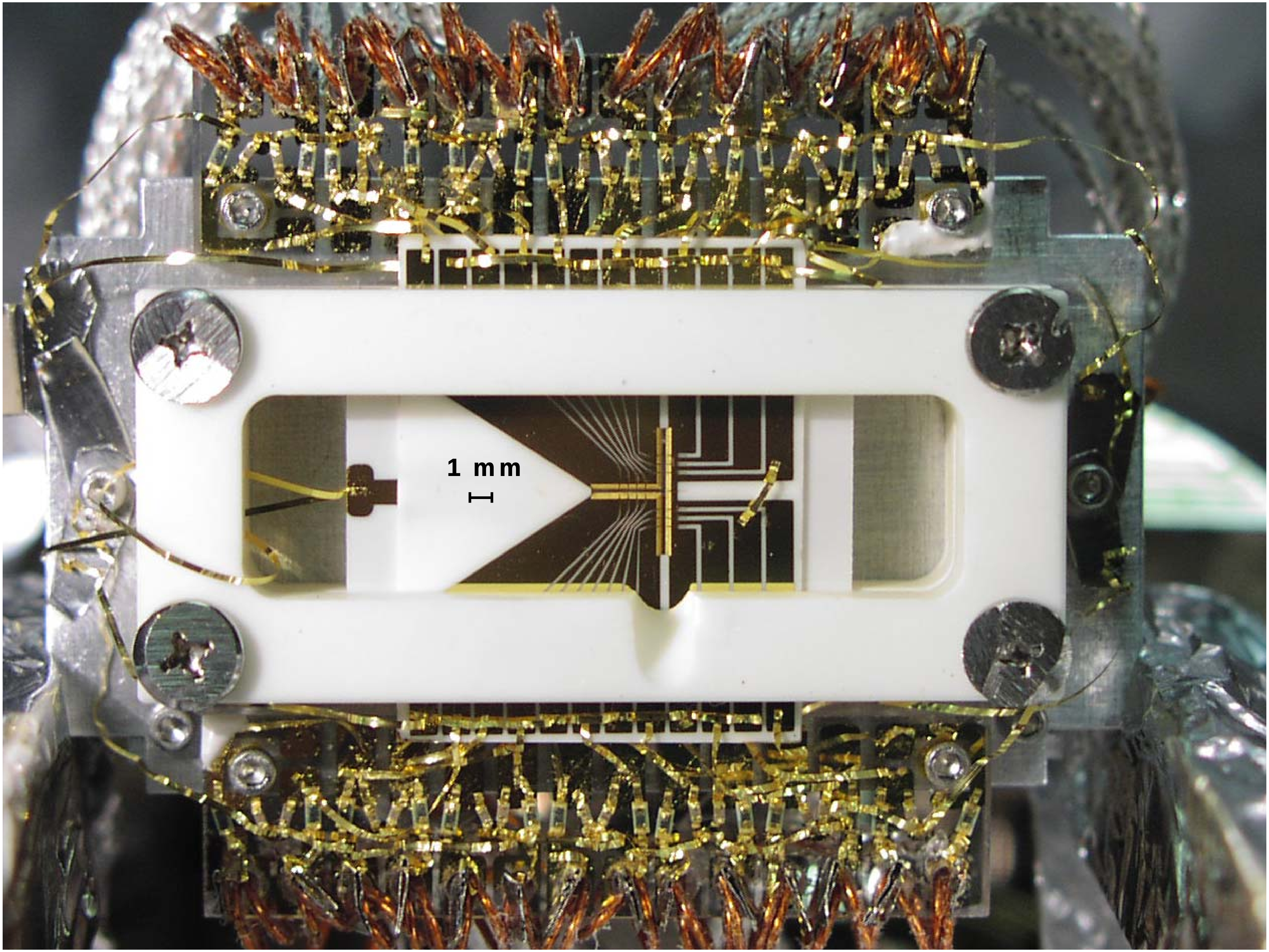
- Modest inhomogeneity---J at edges 90% of J at center. Four-site chain. Tune ratio of Jz/Jx

When Sz is a good quantum number

- The measurement of the z-component of spin for each ion gives the quantum phase with just ONE measurement for the given parameters.
- By measuring the probability to see given configurations in the GS wavefunction, one can actually construct the probabilities for each component, and by partially measuring the phases and using variational calculations, we can approximately reconstruct the many-body wavefunction!

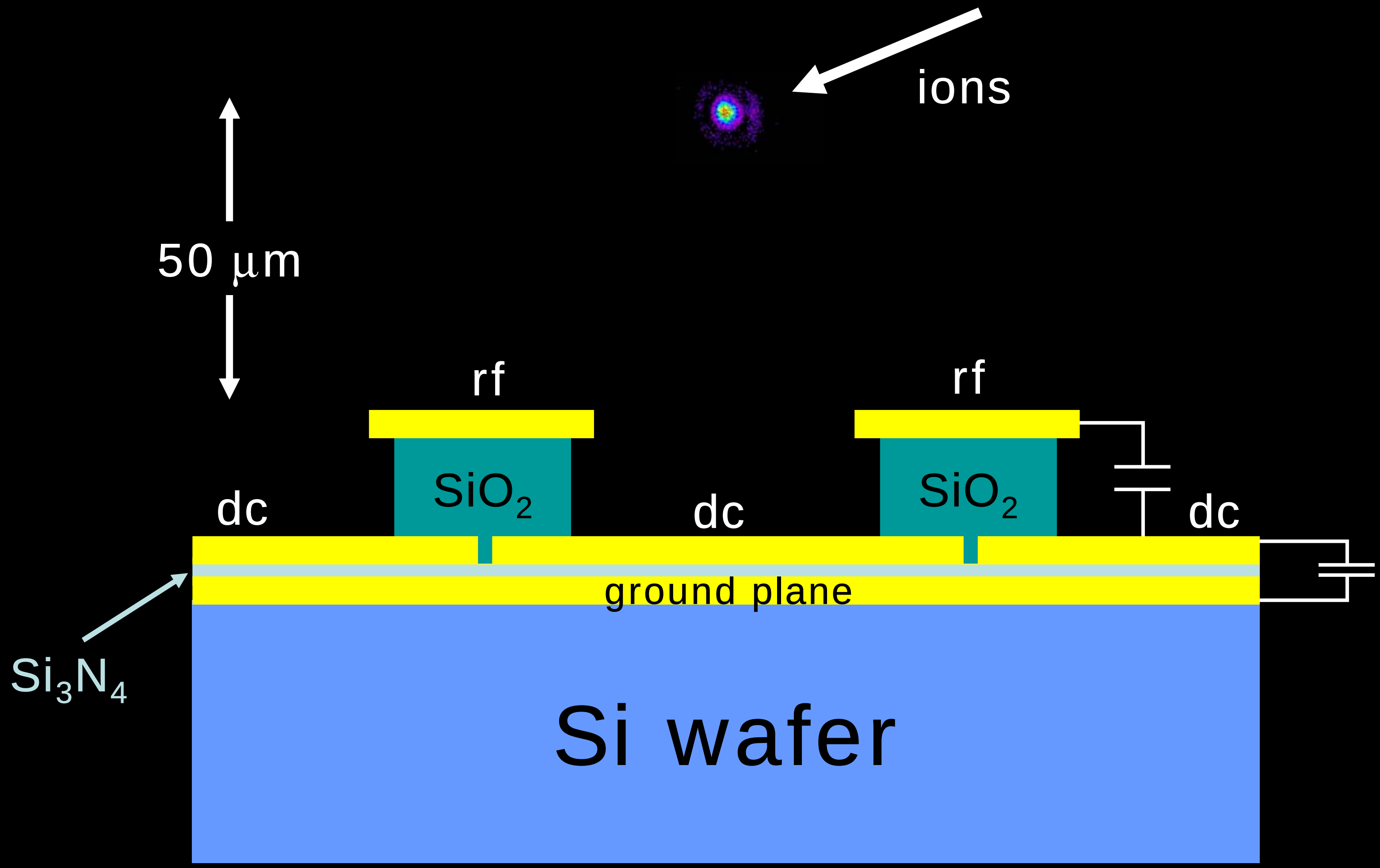
When Sz is not a good quantum number

- Need to measure the probabilities of the different components and map out the contours of constant AVERAGE Sz. Expect the transitions to be rapid when Sz is almost a conserved quantity, allowing phase diagrams to still be constructed.
- The wavefunction can continue to be reconstructed to be used for measuring other aspects of the system (such as correlation functions, energies, etc.)


1 mm

Si/SiO_2 "surface" trap

R. Slusher, Lucent/GaTech



Experimental/theoretical issues

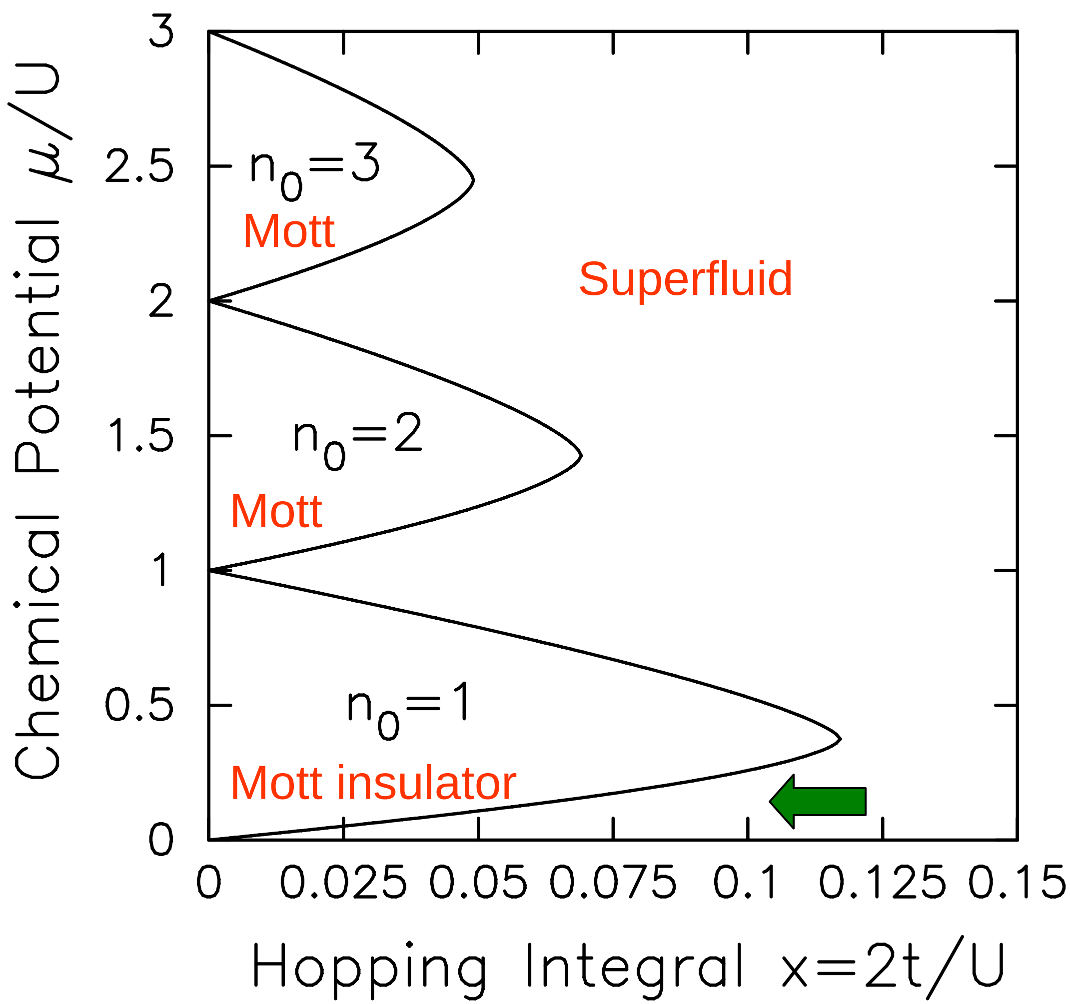
- Temperature effects are not important, since the couplings can be chosen to be large and the system is isolated.
- Inhomogeneity can be corrected via perturbation theory if the wavefunction can be successfully reconstructed.
- Conventional calculations can be performed at T=0 up to about 20 lattice sites exactly.
- The stroboscopic ansatz can be tested by performing time-dependent numerical studies.

Two-dimensional Bose Hubbard model

What is already known from theory

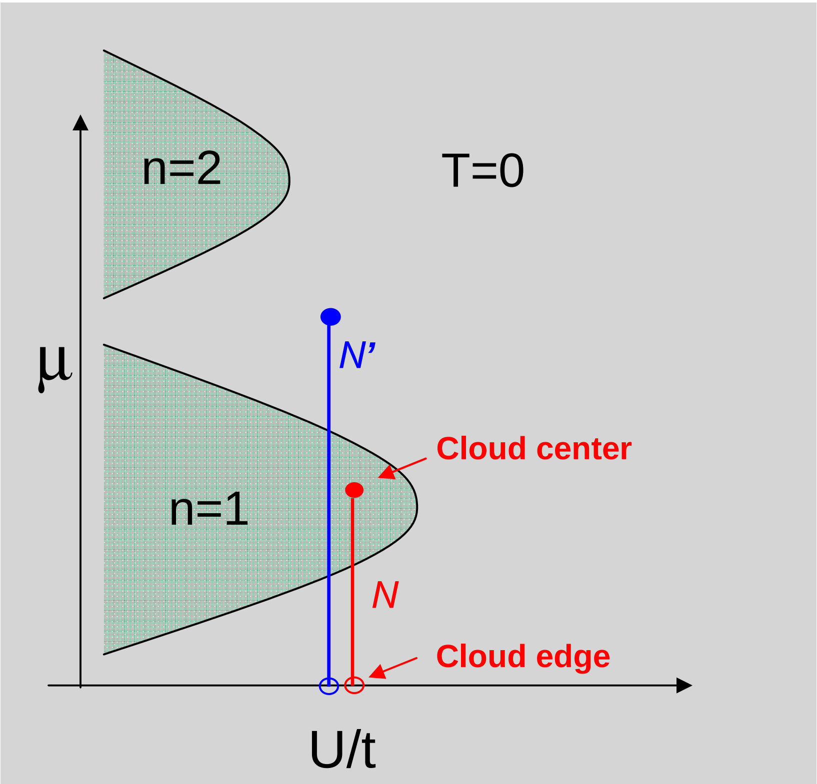
- Extrapolated strong-coupling expansions produce a T=0 phase diagram for the homogeneous (bulk) two-dimensional square lattice with an accuracy better than 1%.
- Quantum Monte Carlo calculations show the size of the Mott lobes is insensitive to temperature for temperatures on the order of U/5 or t/5.
- QMC in a trap shows the wedding cake structure and can be used to benchmark the local density approximation.

Homogeneous phase diagram



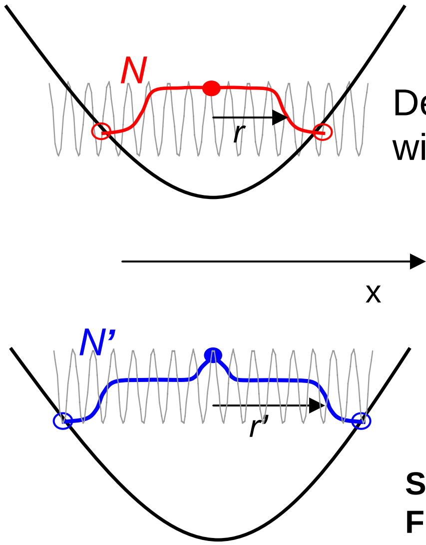
Two-dimensional homogeneous phase diagram at T=0 calculated with a constrained extrapolated strong coupling expansion to third order in t/U. The accuracy is very high to the left, and the critical point is accurate to better than 1%.

Inhomogeneity effects



Within the local density approximation (LDA)

$\mu = \mu(\vec{r})$, set by the trapping potential



See, e.g.
Folling et al.
PRL 97, 060403 (2006)

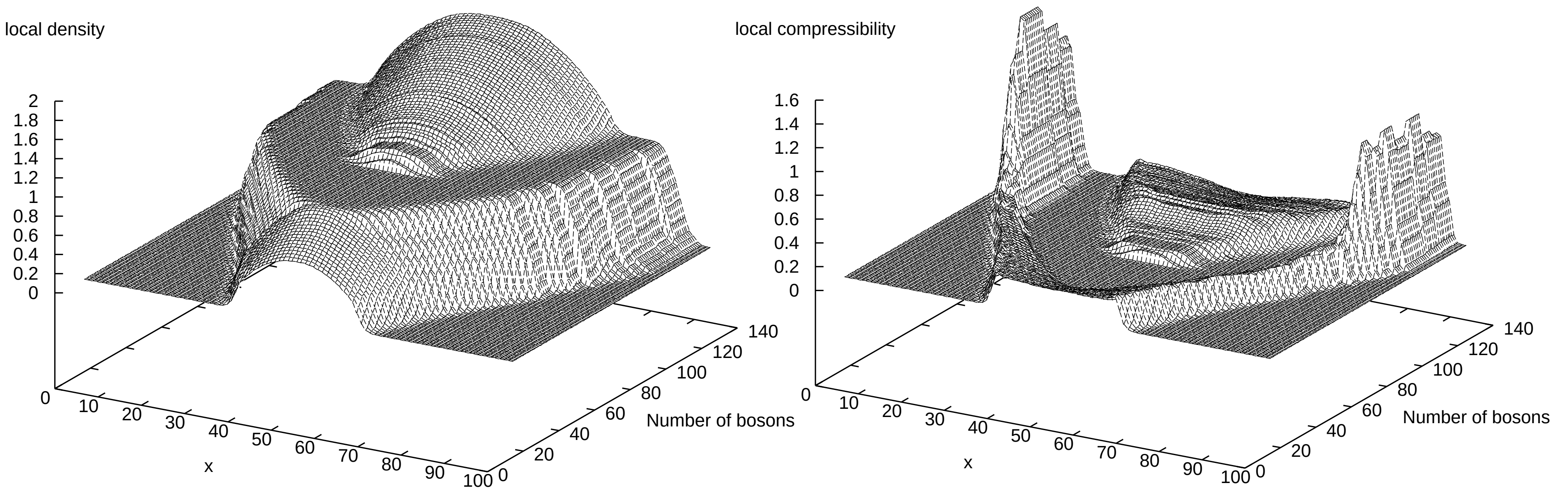
Inhomogeneity not necessarily bad, (in fact it is often useful)
Need:

to satisfy LDA
+
local probes

or

Homogeneous system

“Wedding cake” structure in a trap



Comparing density profile versus compressibility showing clear surrounding of the Mott phase by the superfluid. This image shows how results vary with different total particle number.

Local density approximation ansatz

- Phase constrast imaging will determine the size of the N=1 Mott phase lobe.
- Using the precise shape of the trap, we can extract the critical chemical potential for the phase transition.
- This is then the measured value for the critical chemical potential in the bulk phase diagram from the local density approximation.
- Extrapolation to the critical point can be achieved if sufficiently high quality data is available, just like it is done for the strong-coupling expansions.

Accuracy of the local density ansatz

- Using quantum Monte Carlo calculations in a trap, we can carefully quantify the accuracy of the experimental protocol and the accuracy of the local density approximation.
- We calculate the density profiles of the wedding cake structure on a 2-d lattice in the same trap as used in experiment, varying the filling and the temperatures.
- We extract the Mott phase lobe location as a function of t/U, extrapolating to the critical point, and compare to the homogeneous phase diagram. This will show the ultimate accuracy that can be attained without any numerical postprocessing.

Experimental probes

Example probes:

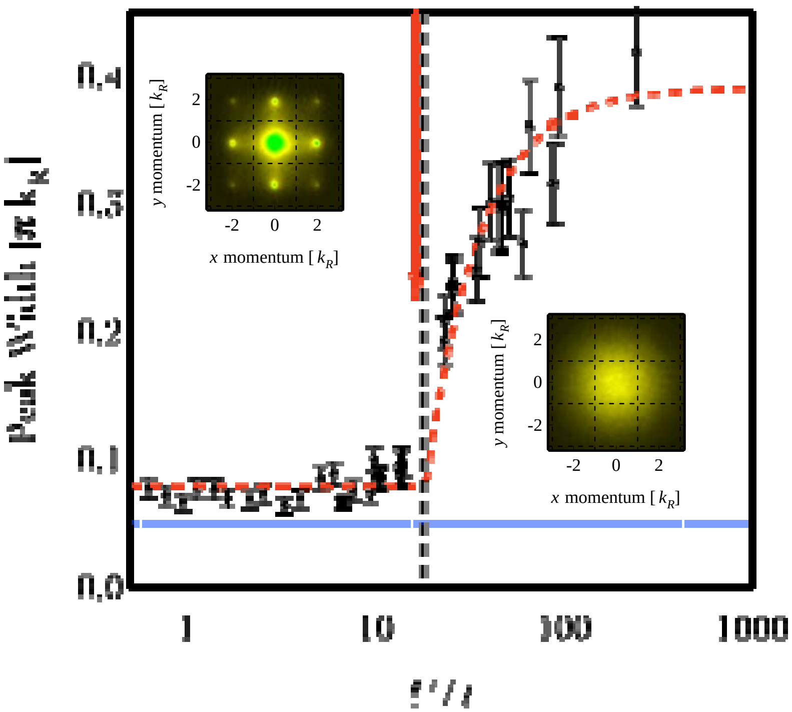
- **Condensate fraction (momentum dist.)**
- **Superfluidity (transport)**
- **Compressibility (density distribution)**

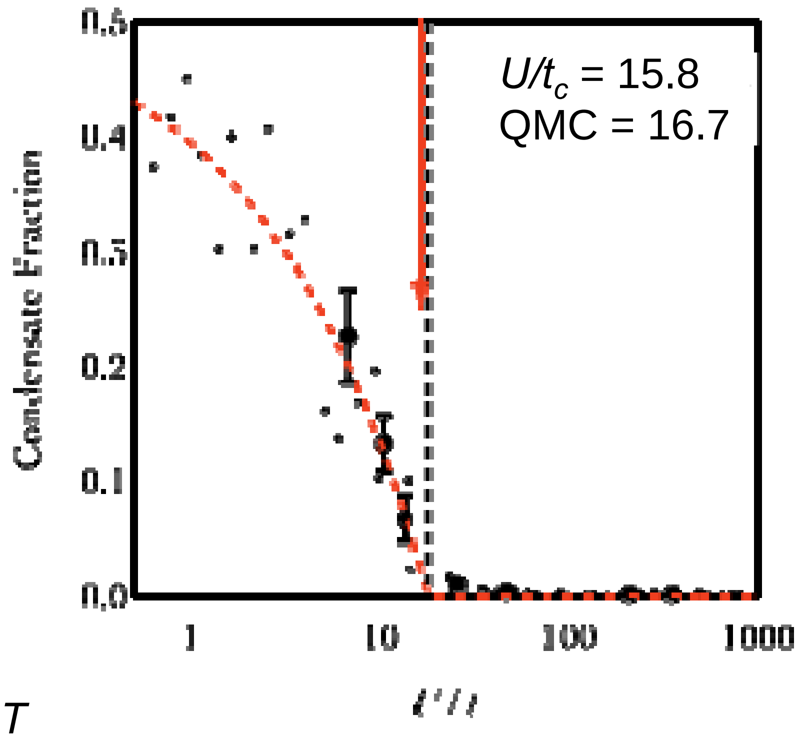
At zero temperature these probes give the same result.

Each of these approaches has different issues to be solved.

All are susceptible to finite temperature effects

Example: superfluid density (NIST)



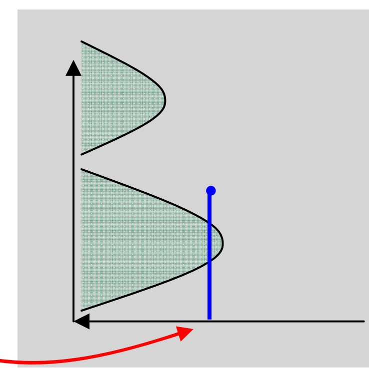


NIST

Despite numerous issues (temperature, inhomogeneity)

⟶ gives a >10% measure of single point U/t_c

(T ~ 0.5 t in the superfluid)



Conclusions

- Described two thrusts in our OLE project
- Numerical methods will allow precise determination of the different phase diagrams needed for benchmarking.
- Theory will work to determine effects of temperature and inhomogeneity to maintain the <10% confidence level required for the demonstration.
- Theory will also work on various different postprocessing schemes to remove effects of inhomogeneity, or to extract additional information about the quantum many-body state than what can be directly measured in experiment.