A simple description of the insulator-superfluid transition in the Bose Hubbard model

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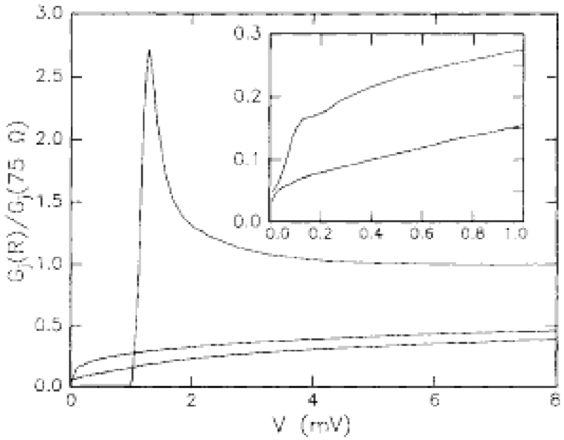
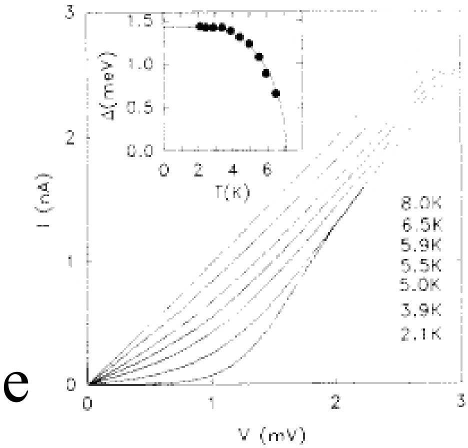
Motivation: granular thin-film superconductors

Dynes et al. PRL **69**, 3567 (1992);
Bi films have the **sc gap go to zero** at the superfluid-insulator transition.



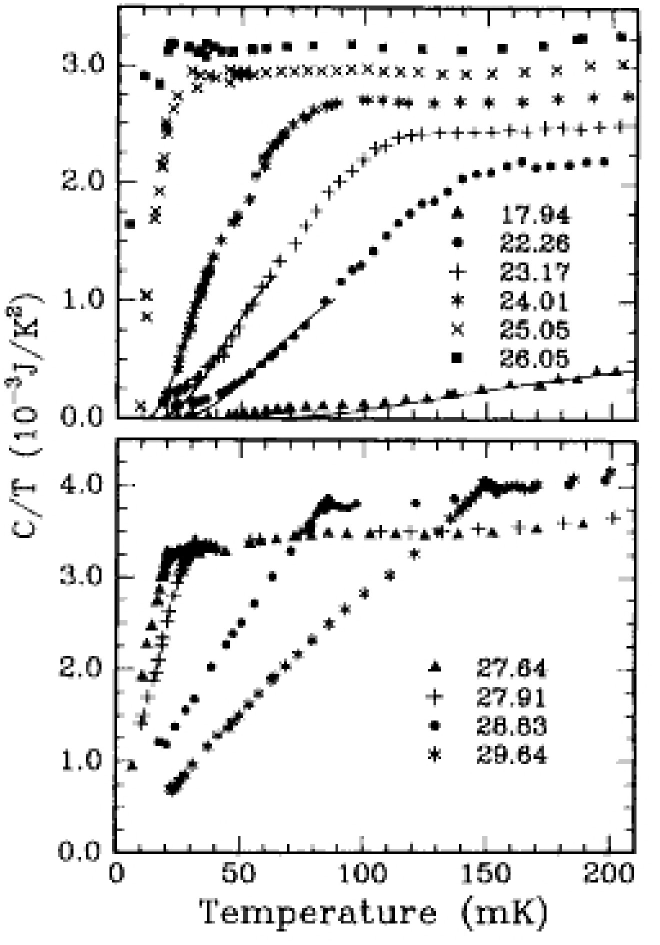
But, Dynes et al. PRB 49, 3409 (1994);
shows that in Pb films, the sc **gap remains essentially at the bulk value** at the superfluid-insulator transition.



- Local pairing, but no global phase coherence

He^4 films on vycor or aerogel

- *Reppy, et al. PRL* ***75****, 1106 (1995)* show that in the presence of disorder, Bose systems can still have an excitation gap at low temperature (Cv/T drops rapidly at low-T).
- This implies that the Bose glass does not always form in disordered cases, as suggested by Fisher, Weichman, Grinstein, and Fisher.

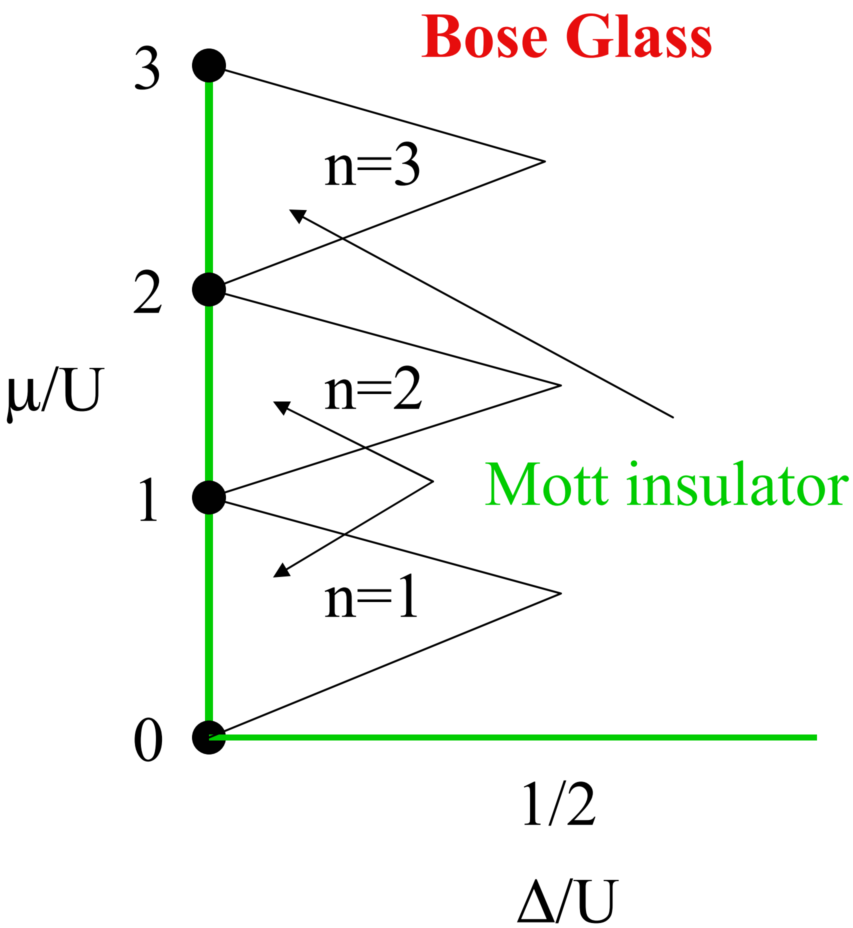


Bose Hubbard model

$$H = -\sum t_{ij} a_i^* a_j + \sum \varepsilon_i n_i + \frac{1}{2} U \sum n_i (n_i - 1) - \mu \sum n_i$$

- *Mobile soft-core bosons interacting with an on-site Coulomb interaction---the bosonic version of the Hubbard model (magnetic field enters in the phase of t_{ij})*
- **Granular superconductors**
- **He^4 in porous materials**
- **Josephson junctions (vortices: E_J~U, E_C~t; Cooper pairs: E_J~t, E_C~U)**
- **Optical lattices and ultracold atoms**
- **Calculational methods: QMC, DMRG, RG, PERT THEORY**

t=0 phase diagram

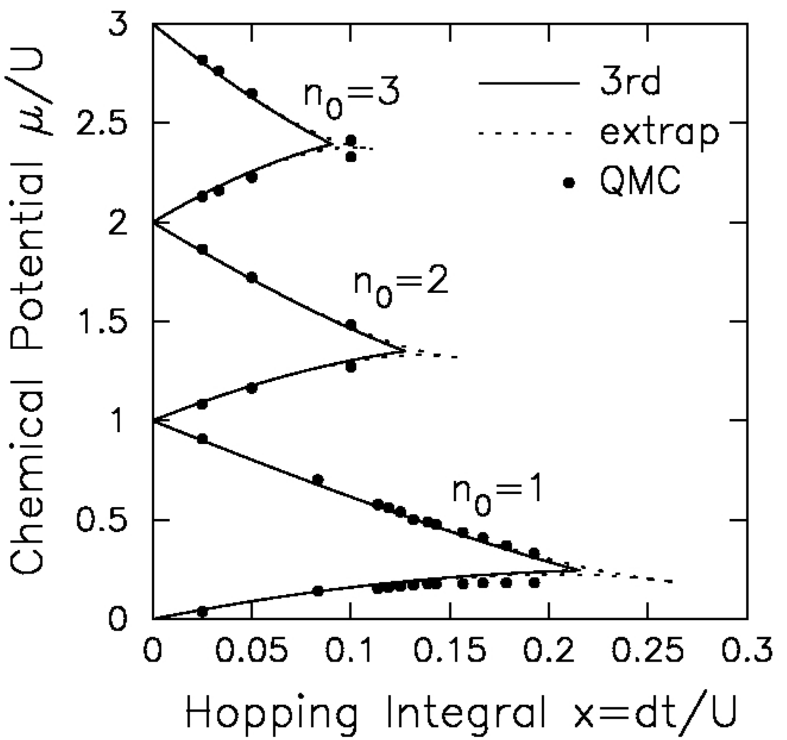


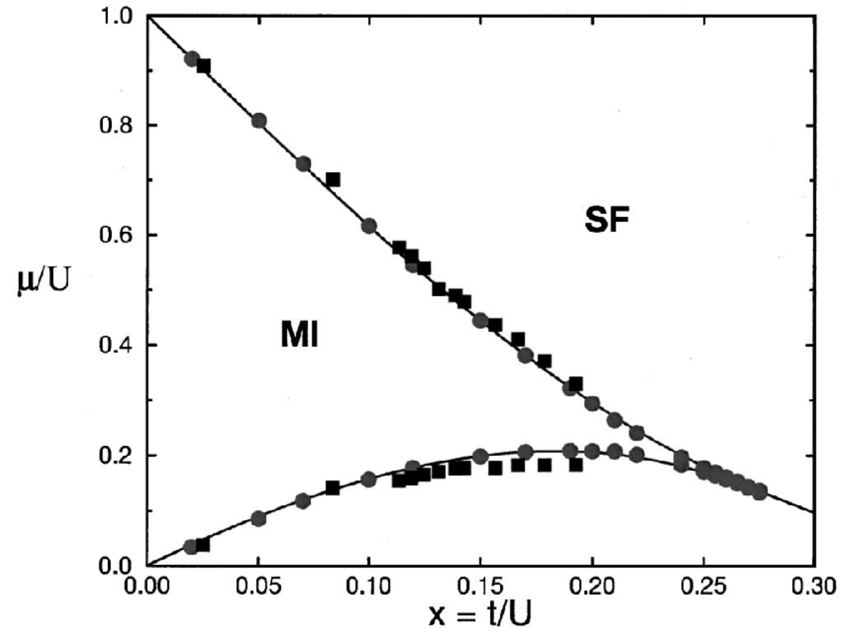
- Pure case:
  $E(n)=Un(n-1)/2-\mu n$
  $E(n+1)-E(n)=Un-\mu$
- Disordered case:
  (bounded disorder $|\varepsilon|<\Delta$)
  $E(n+1)-E(n)=\varepsilon_i+Un-\mu$

Perturbation theory in t

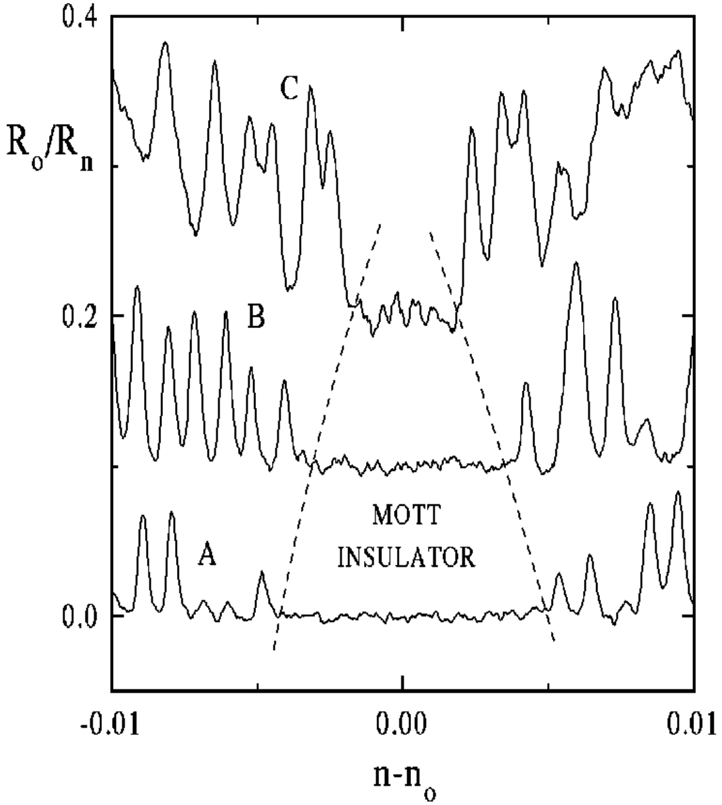
- The Mott phase has **exactly** n bosons per site. No charge fluctuations are allowed, so the system is an **incompressible fluid with a gap**.
- Consider the excitation energy to the (defect) state with **one extra or one fewer boson.**
- As the gap vanishes, there is a **transition to a compressible phase** (Bose glass or superfluid). Easy to calculate, just equate the energies.
- Perform perturbation theory in single-particle operator “t+ε”.

D=1, no disorder





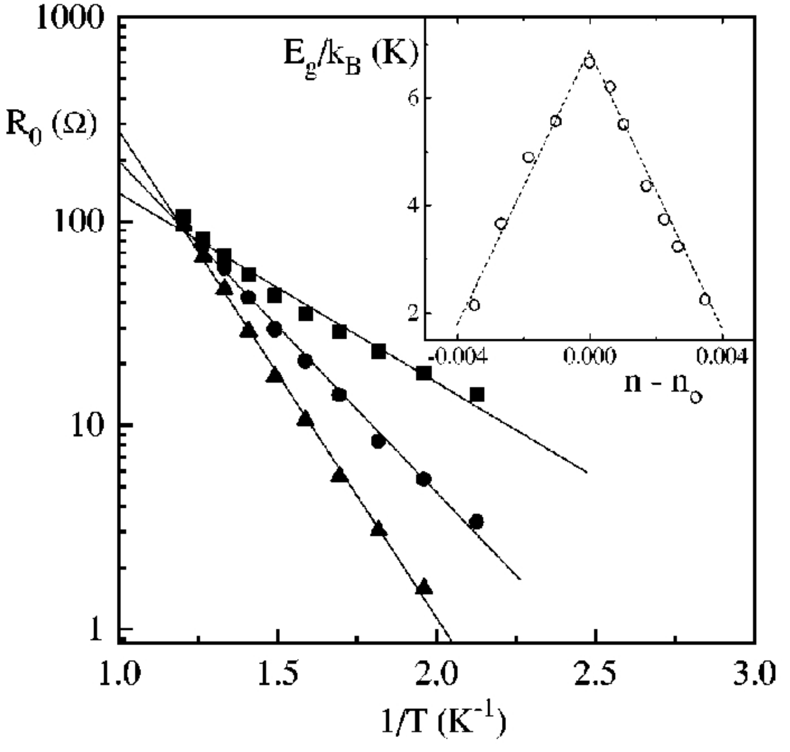
Josephson junction arrays



Data from Mooij's group.

- Experimental data on 1-d Josephson arrays.
- Horizontal axis is analogue of the chemical potential μ.
- Vertical axis is analogue of t/U (curves shifted by value of ratio E_C/E_J).

Josephson junction arrays

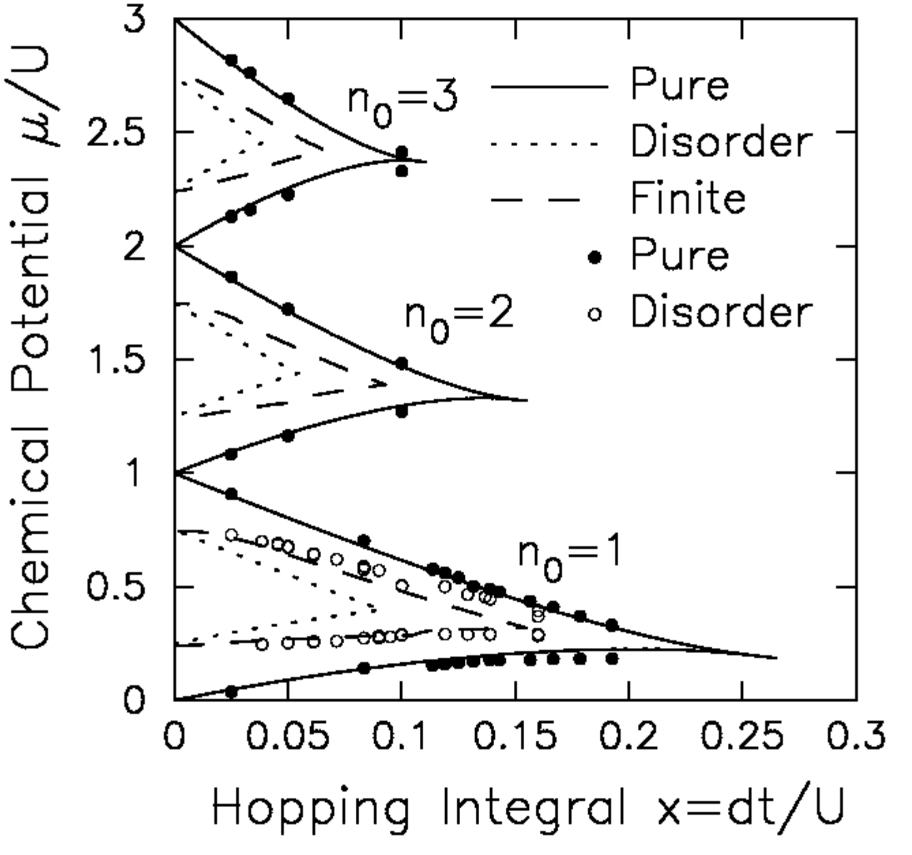


Data from Mooij's group.

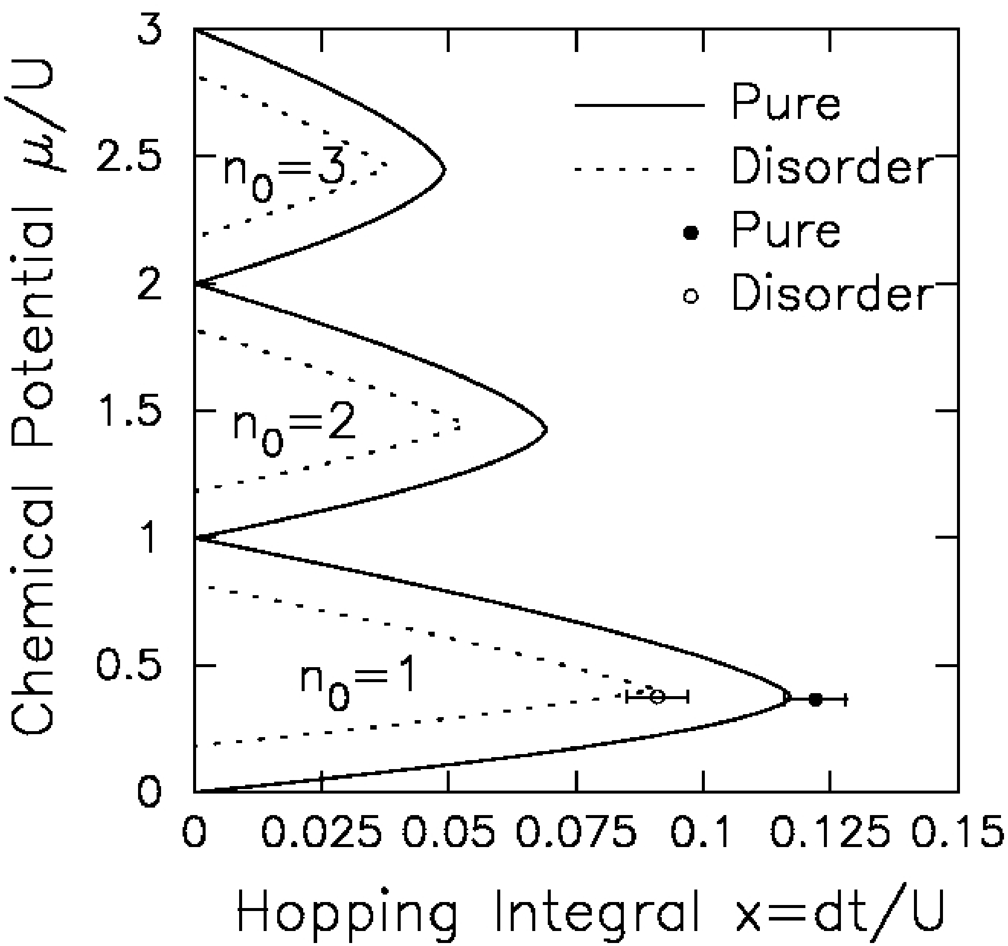
- Effective gap energy can also be extracted from the experimental data.
- Note how the results have a cusp-like feature just like the perturbation theory showed.

D=1, disordered

- Rare regions indicate that the critical behavior at the tip disappears for any value of disorder.
- Finite-size effects are large since these systems have no rare regions.
- Results strongly suggest that the Mott phase is completely surrounded by the Bose glass phase.

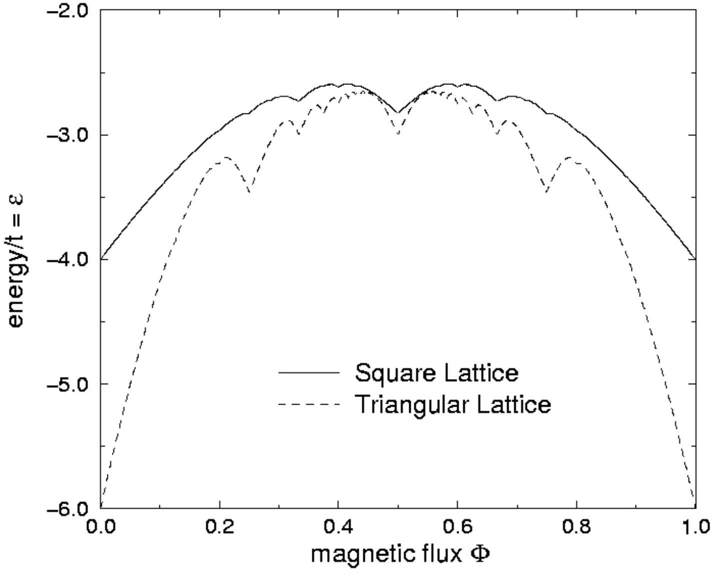


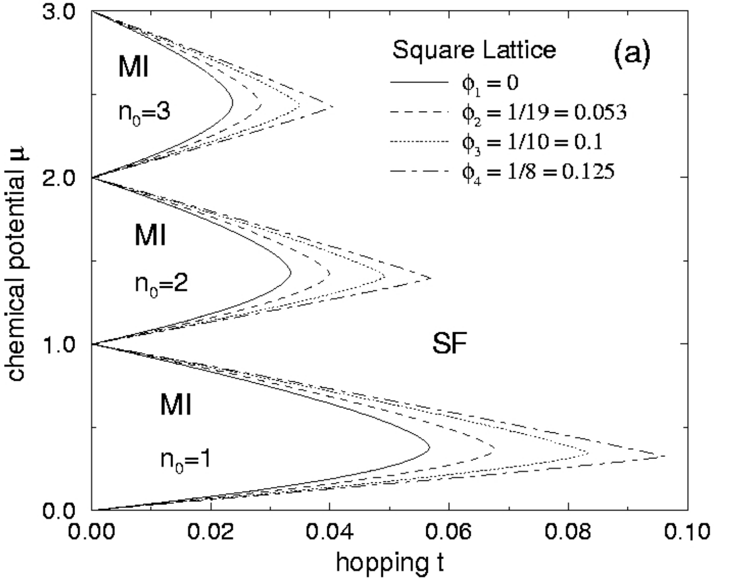
2D pure case and disordered



- Now the tip of the lobes in the pure case have a power law dependence.
- Disorder once again will break the "critical behavior" at the tip and produce a discontinuity in the slope.

2D case, magnetic field



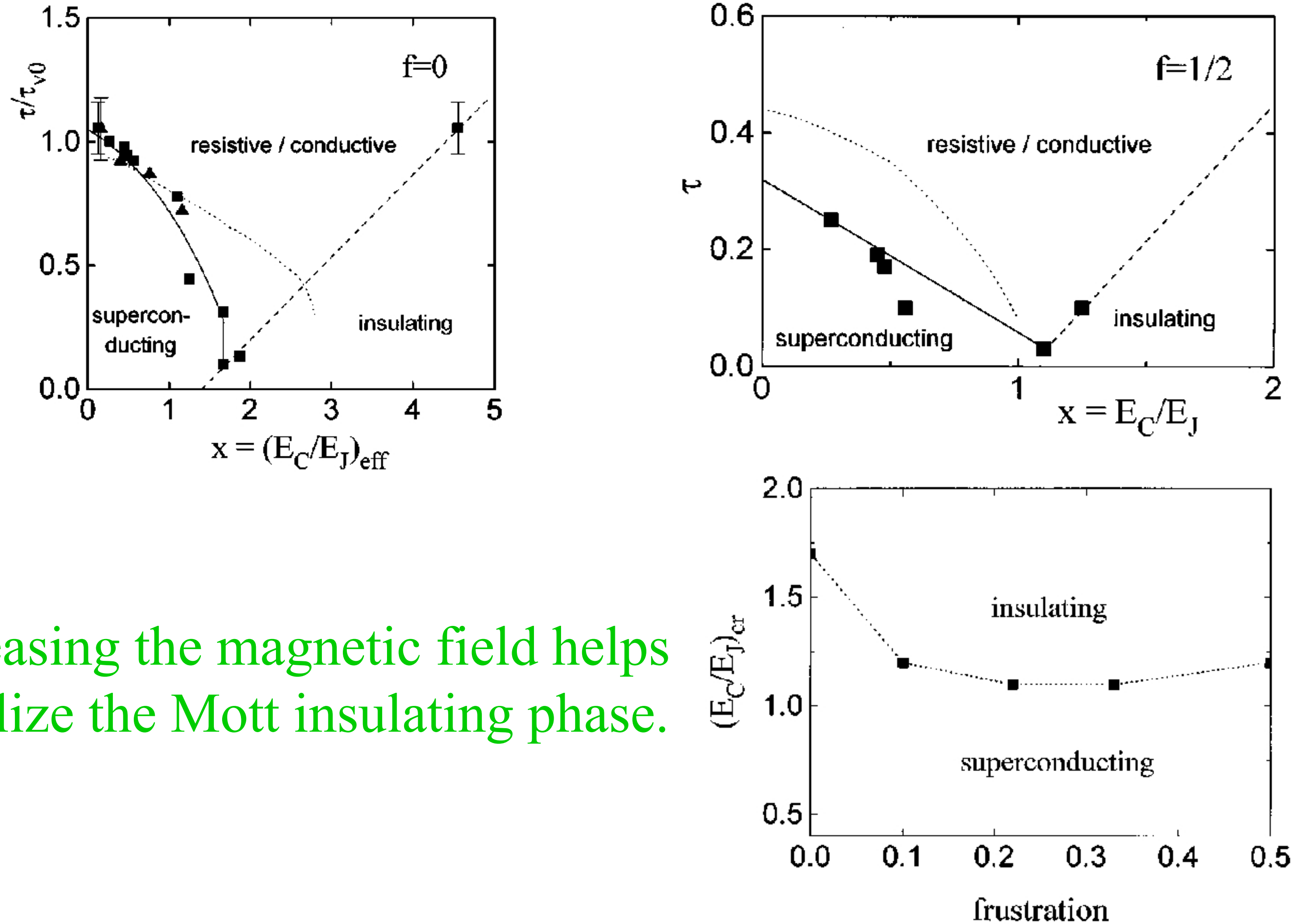


•Hofstadter problem is needed to determine the first-order degenerate perturbation theory.

•The presence of a magnetic field stabilizes the Mott insulator and increases the size of the Mott lobe. As the field increases the perturbation theory breaks down . The tip is expected to be “first-order” here as well.

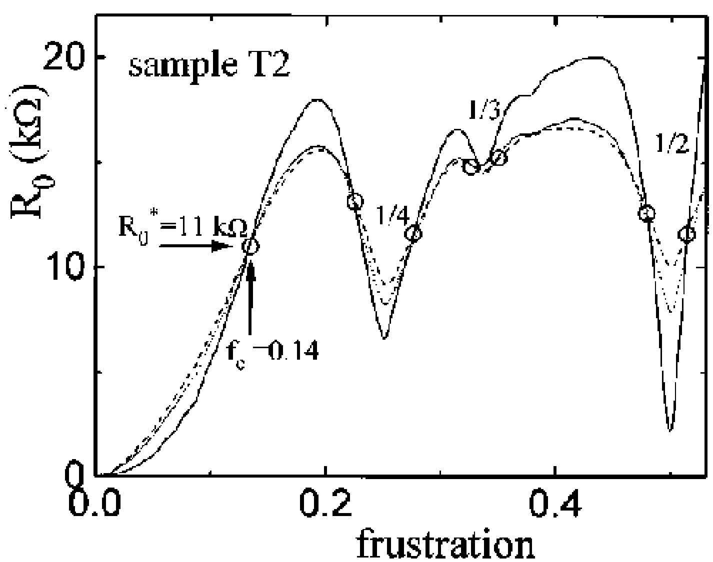
Josephson junctions, 2D

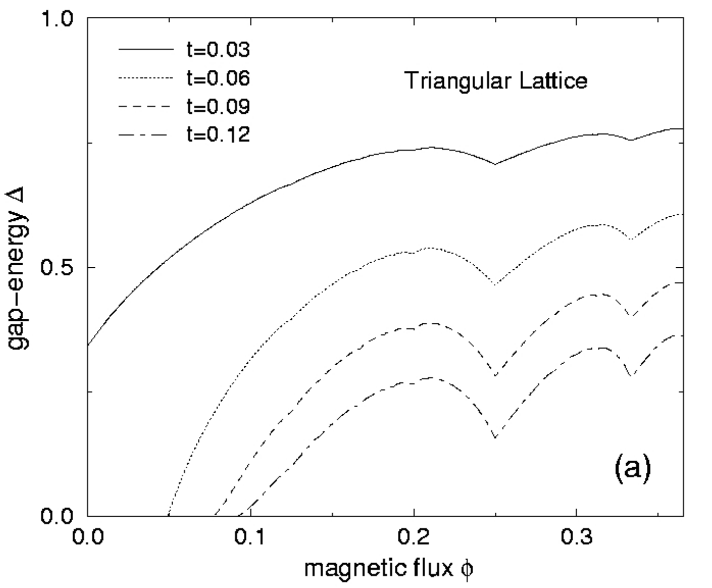
- Now we use the Cooper-pair picture rather than the vortex picture.



Increasing the magnetic field helps stabilize the Mott insulating phase.

Josephson junctions, 2D ct'd

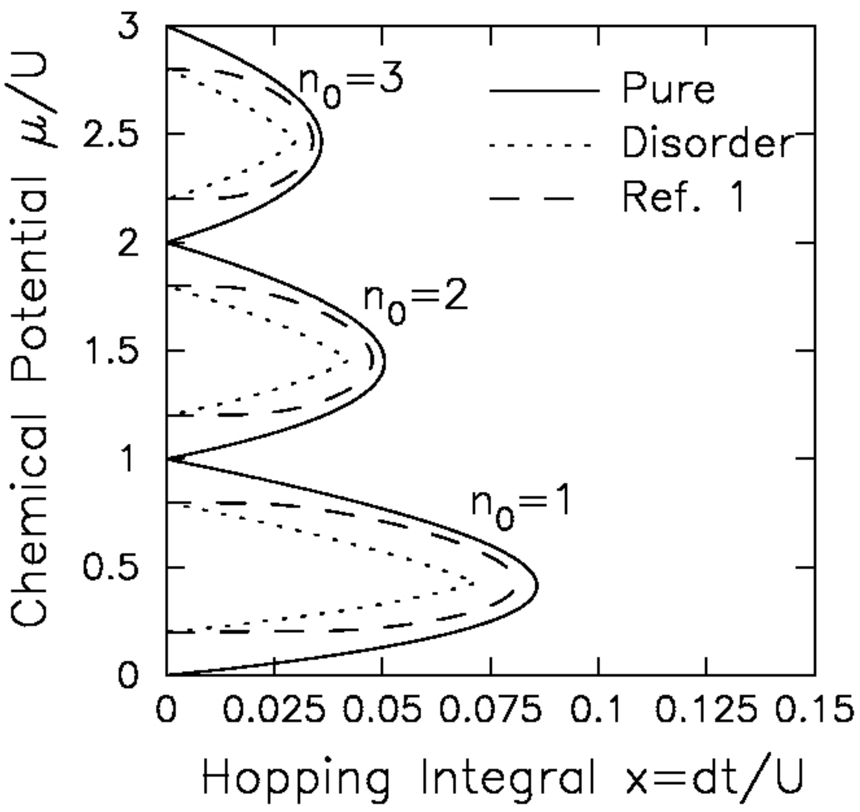




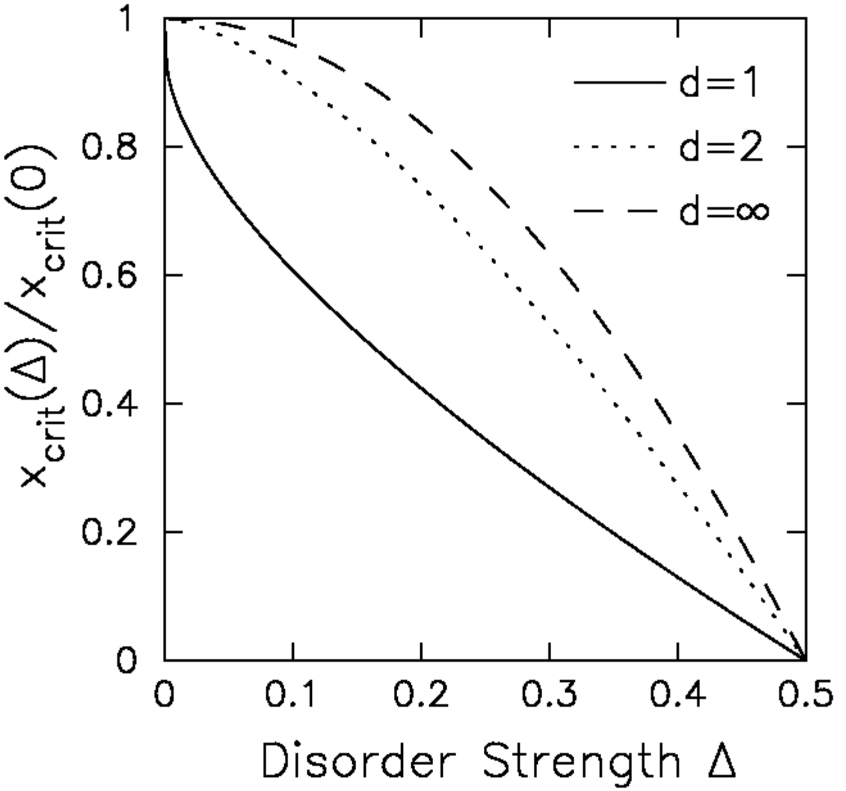
Excitation gap as a function of magnetic field shows similar behavior in theory and experiment.

Exact solution as d→∞

- Must scale $t=t^*/d$.
- Dashed line is the exact solution for the infinite-range hopping model.
- Region between dotted and dashed line is where the compressibility will be exponentially small.

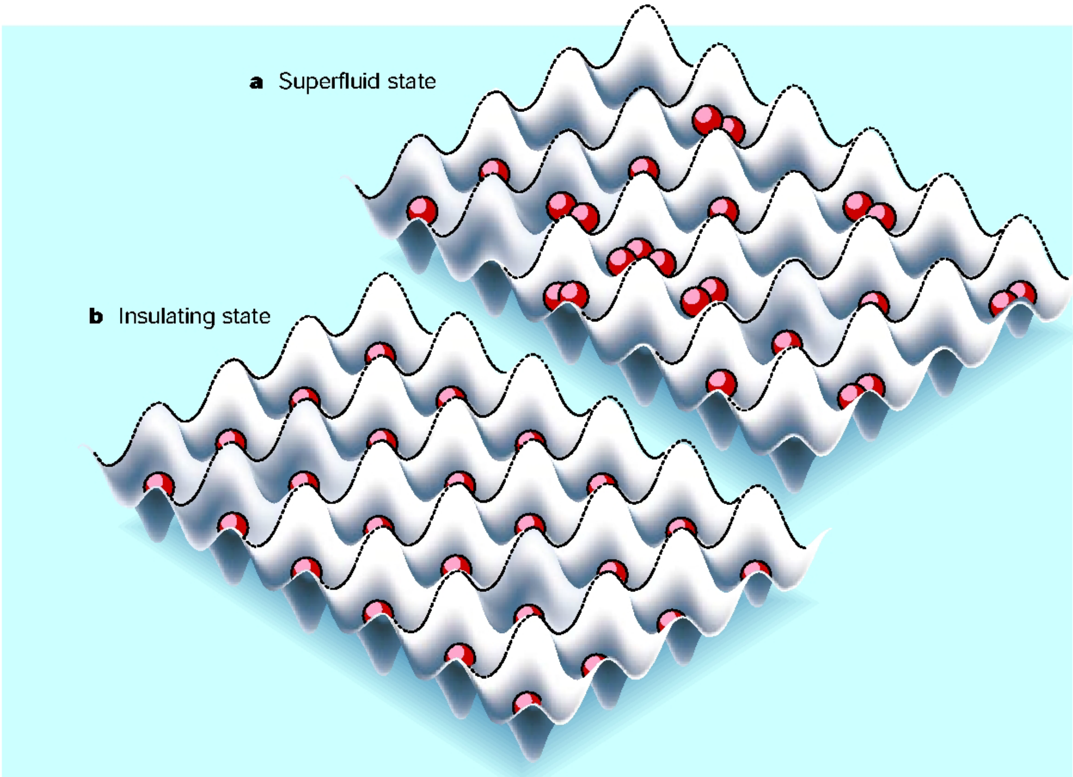


Summary of disorder for all D



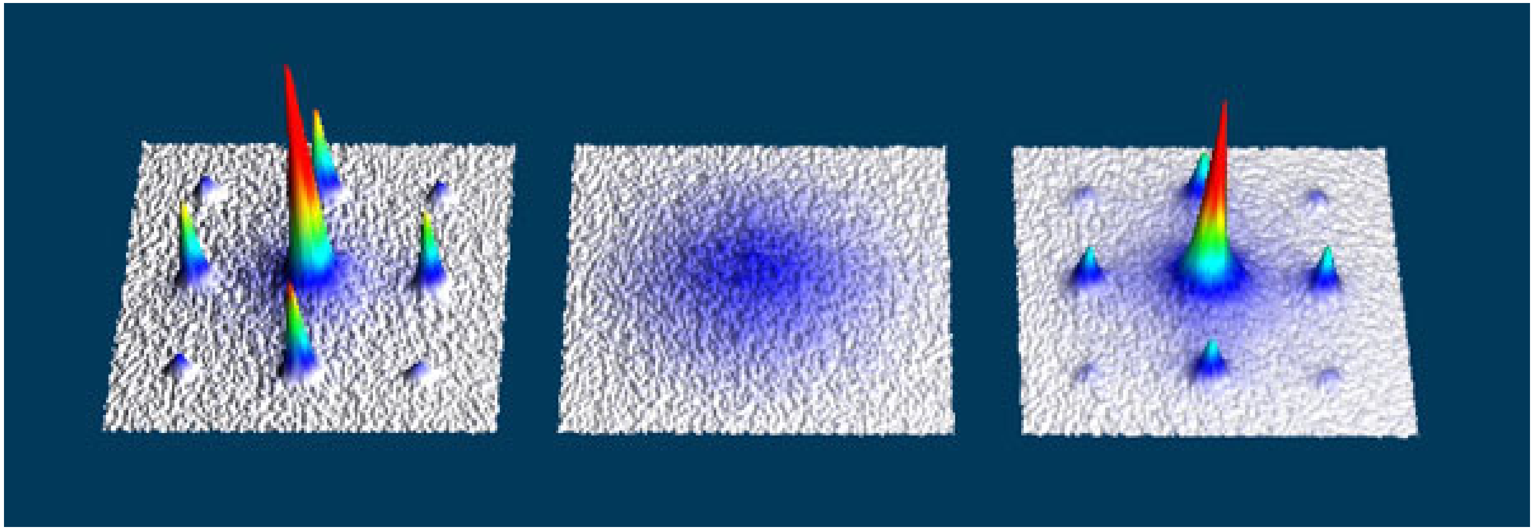
- The dependence of tip location on disorder is strongest in 1d, since the lobe is so sharp at the end.
- Dimensions 2 and higher have similar dependence on disorder (and weaker than 1D).

Optical lattices



- Hänsch et al. have shown how to create a 3D lattice that traps ultracold atoms.
- The lattice is superimposed on the magnetic trap, which makes the system inhomogeneous overall.

Superfluid-Insulator transition



- Coherence peaks indicate superfluidity (or at least delocalization through the lattice).
- The broad structure indicates the localized phase.
- By adjusting t/U, the superfluid and insulating phase can be entered again and again.

New frontiers (theory)

- Accurate calculation of the Bose-glass-superfluid transition is lacking.
- Does disorder immediately remove the critical behavior at the tip of the Mott lobe, or is there a critical disorder where it changes character.
- Inhomogeneous systems including a trap potential---is there a real Mott transition, and how is it characterized.

New frontiers (experiment)

- Can the analogue of magnetic fields be introduced into optical lattices?
- Can controlled experiments with disorder be performed?
- Can a lattice be constructed without a harmonic trap?
- Can one control longer range interactions and look for novel behavior like supersolids.