Erratum: Tuning a Josephson junction through a quantum critical point [Phys. Rev. B 64, 054511 (2001)]

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We have corrected a problem in our computational code that determined the normal-state resistance. In general, we find a reduction of R_n when the correlations are large and there is more than one plane in the barrier. The reduction of R_n affects four figures from the original paper, and affects the conjecture about "intrinsic pinholes."

We discovered that our calculations had not fully converged for the interacting density of states and self energy for small frequencies. This created numerical errors in the calculation of the resistance via the Kubo formula. The error has now been corrected and all resistances recalculated. The error does not affect the superconducting properties at all since they were calculated with a different imaginary-axis code.

We show a corrected plot of the Josephson critical current, the normal-state resistance R_n , and the characteristic voltage I_cR_n for the single plane and two-plane barriers. The plots are on a semilogarithmic scale. The main difference is in the characteristic voltage I_cR_n in Fig. 4(c). The optimization now occurs at the ballistic metal limit of $U\rightarrow 0$, where the I_cR_n product approaches the product of the bulk critical current times the Sharvin resistance, which is 0.287t/e= $1.45\Delta/e$. As U_{FK} increases, the characteristic voltage decreases and becomes flat as expected by the Ambegaokar-Baratoff limit. The results for the two-plane barrier are modified significantly. The reduction of I_cR_n is more dramatic for moderate scattering, and as we move into an insulating barrier, we also reproduce the Ambegaokar-Baratoff result, and the unphysical increase with U_{FK} has disappeared.

Next we show the corrected figure for the moderately thick barrier (with $N_b = 5$). The critical current, normal-state resistance, and characteristic voltage are plotted in Fig. 9 for $N_b = 5$ (diamond). The characteristic voltage has interesting behavior. Starting at a value about 20% less than the Ambegaokar-Baratoff limit in the metallic regime, the voltage initially decreases with correlation strength, then has a rapid increase starting at the metal-insulator transition, reaching a maximum near the Ambegaokar-Baratoff result (up to the values of U_{FK} that we can safely determine the I_cR_n product). Junctions in this correlated regime do see an enhancement of the characteristic voltage on the insulating side of the metal-insulator transition, but the enhancement is at most only 20–30% higher than in the thin tunnel junctions.

The thick barrier junction $N_b = 20$ has the most significant corrections. We find that the calculations become untrustworthy when U_{FK} becomes too large. We show results up to $U_{FK}=6$, but it is possible the I_cR_n product is too large there, due to an overestimate of the critical current I_c , as can be inferred by the slight upturn in the I_c data in panel (a).

The critical current, normal-state resistance, and characteristic voltage appear in Fig. 11. One can see the metalinsulator transition clearly in the I_c curve. Within the metal, the critical current has an exponential dependence on U_{FK} ; within the insulator it has a different exponential dependence. In the transition region, it decreases most sharply. The resistance shows the expected behavior as well. One can clearly see the metal-insulator transition as the region where the conductivity changes its functional dependence sharply. Note, however, that the characteristic voltage is severely affected by the correlations. It has an initial decrease, as seen for thinner junctions, as we introduce scattering into the metal, but the behavior may change at the metal-insulator transition, and be converted into an upturn, but it is difficult to gauge the accuracy of our results when U_{FK} becomes too large.

We summarize the corrected characteristic voltage data as follows. The characteristic voltage is limited in the metallic regime by the bulk critical current of the superconductor multiplied by the junction resistance for a clean barrier (the so-called "planar contact" limit). This value is approximately $1.31\Delta/e$, which is about 8% smaller than the Ambegaokar-Baratoff result for an insulating barrier. As the



FIG. 4. (a) Critical current, (b) normal-state resistance, and (c) characteristic voltage of the Josephson junctions as a function of the Falicov-Kimball interaction within the barrier. The circular symbols are for $N_b = 1$ and the squares are for $N_b = 2$. Note how the critical current decreases and the junction resistance increases as expected, and how the characteristic voltage does not depend too strongly on the correlation strength. The dependence on correlation strength for the bilayer junction is stronger than for the thin junction. The dashed line in (c) is the Ambegaokar-Baratoff prediction.



FIG. 9. (a) Critical current, (b) normal-state resistance, and (c) characteristic voltage of the $N_b = 5$ (diamond) junction. The metalinsulator transition can be seen in the critical current (a) and (more easily) in the resistance (b), as the regions where the slope of the curves changes most dramatically. In the strongly correlated insulating regime, we find the exponential decay of the current (and increase of the resistance) has a different slope than in the correlated metal regime. The characteristic voltage (c) has complex behavior: it first decreases in the metallic regime, then has a rapid increase starting at the metal-insulator transition (the Ambegaokar-Baratoff prediction is the dashed line).



FIG. 11. (a) Critical current, (b) normal-state resistance, and (c) characteristic voltage of the $N_b = 20$ (triangle) junctions. Note how the thick junction behaves much like the bulk material. The metalinsulator transition can be clearly seen in the critical current and in the resistance, as the regions where the slope of the curves changes most dramatically. In the strongly correlated insulating regime, we find what appears to be an upturn of the I_cR_n product, but it is difficult for us to estimate the error in the $U_{FK}=6$ data point. For larger U_{FK} values, we are unable to find accurate results for our calculations. The dashed line is the Ambegaokar-Baratoff prediction.



FIG. 12. Characteristic voltage plotted versus the inverse of the effective thickness of the barrier on a log-log plot. Using the correlation length extracted from the fit, allows us to plot the characteristic voltage against a measure of the Thouless energy $E_{\rm Th} = 2 \pi k_B T \xi_b^2/N_b^2$. Such a plot should show scaling behavior, according to the quasiclassical theory; we find this to be approximately true for the metallic junctions ($U_{FK}=2$, circles; and $U_{FK}=4$, squares), but the correlated insulating barrier has a different dependence on the barrier thickness. Note the start of the crossover in the R_n from tunneling-like behavior, where it increases exponentially with thickness, to a linear-scaling regime, as expected in the bulk "Ohmic-transport" limit.

correlations increase, I_c decreases to be much below the bulk critical current of the junction, and R_n increases. The characteristic voltage has a rich behavior. For the thin junction $(N_{h}=1)$ it is maximized in the ballistic metal regime, is depressed as scattering is introduced, and becomes constant for the insulator. As the thickness increases to $N_{b}=2$, we continue to see the initial depression, but it is stronger here, and then is followed by the constant value again in the insulator. For barrier thicknesses on the order of the correlation length $(N_b = 5)$, the behavior is more complex. The voltage initially decreases with correlation strength, then has a rapid rise at the metal-insulator transition; we believe it will achieve a maximal value for some large value of U_{FK} but we cannot push our calculations beyond $U_{FK} = 10$. The Ambegaokar-Baratoff picture does not hold here, as $I_c R_n$ depends on the bulk insulating gap in this regime. Finally, in the thick junction regime $(N_b = 20)$, the characteristic voltage has an interesting dependence on the correlation strength. It also shows the depression as scattering is introduced, and it appears to have a turn-over to increase once the metal-insulator transition is passed, but we are unable to push our calculations beyond $U_{FK}=6$ here. Note that the characteristic voltage must decrease when the correlated insulator is made thick enough, because the critical current decreases exponentially with the thickness, but the resistance will grow only linearly with the thickness as one enters the bulk "Ohmic scaling" region. The conclusion that can be drawn from this is that one requires a careful tuning of the thickness of the barrier, the proximity to the metal-insulator transition, and the operating temperature to optimize the properties of a junction.

In the corrected Fig. 12, we plot the characteristic voltage (in units of Δ/e) versus the ratio of the barrier coherence length (determined from the fit of the critical current and

equivalent to the Thouless length) to the barrier thickness. The results for metallic junctions $U_{FK} = 2,4$ are essentially unchanged. The correlated-insulator results, $U_{FK}=6$, however, show different behavior, remaining flat for a wider range of thicknesses and then starting to turn over at the thickest junction we can perform the calculation at $N_b = 20$. One may wish to conclude from Fig. 12 that correlated insulating barriers are superior to metallic barriers since the parameter ξ_h/N_h can be reduced to much smaller values than in the metallic cases before the characteristic voltage becomes reduced. But such a view is erroneous, because the significantly smaller values of ξ_b for the insulating barriers means that the barrier thicknesses where the characteristic voltage starts to decrease are indeed smaller for the correlated insulator. We are no longer able to say what happens for much thicker junctions. While $I_c R_n$ may decrease faster than expected, and create the "intrinsic pinhole" effect discussed in our original paper, we no longer have sufficient evidence to support that scenario. In the inset, the resistance shows a less dramatic dependence on thickness than seen previously. There is now a smooth crossover from an exponential increase to a more linear scaling, which is expected in the bulk "Ohmic scaling" regime.

In conclusion, we have found the main effect of the corrected values for the resistances to produce results for the characteristic voltage that are more physical. We no longer are able to say whether a thick correlated insulator will develop the appearance of "intrinsic pinholes" because we cannot perform calculations for too thick junctions. Our results do indicate promise in moderately thick junctions with barriers tuned close to the metal-insulator transition, but it does not seem like the enhancement in switching speed would be more than 20-30 %.

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