Superconductor-Correlated Metal-Superconductor Josephson Junctions: An Optimized Class for High Speed Digital Electronics

J. K. Freericks, B. K. Nikolić, and P. Miller

***Abstract*—It has long been conjectured that tuning the barrier of a Josephson junction to lie close to the metal-insulator transition will enhance the switching speed and provide optimal performance. We examine a new class of junctions, so-called SCmS junctions, where the barrier is a correlated metal (or insulator) close to the metal-insulator transition. We show that high I_cR_N products and moderate temperature derivatives of I_c can be achieved when the thickness and metallicity of the barrier is properly tuned. We believe these junctions show promise for the fastest speed digital electronics operation.**

***Index Terms*—Fast switching speed, Josephson junctions, metal-insulator transition, superconducting devices.**

I. Introduction

THE GOAL of digital superconducting electronics is to provide a faster alternative to semiconductor-based technologies. It is well known that semiconductor technology is nearing its physical limitations, and the increase in clock speed of integrated circuits is approaching its fundamental maximum somewhere in the 10–100 GHz range. Low-temperature superconducting electronics offers an interesting alternative, since the fundamental limitations there lie more in the 100 GHz to 1 THz range (and perhaps even higher for higher temperature superconductors like MgB_2 or the cuprates). One of the challenges with low-temperature superconductor technology has been the difficulty in demonstrating significant increases in speed over semiconductor technology, for chips that perform complex operations (like A/D converters or microprocessors).

Superconducting electronics are based on the Josephson effect [1]. It was originally proposed in tunnel junctions, which consisted of superconductor-insulator-superconductor (SIS) sandwiches with ultrathin I layers. The theoretical description of these junctions appeared soon thereafter by Ambegaokar and Baratoff [2]. The problem with SIS junctions is that they often have a hysteretic (double-valued) current-voltage characteristic, and can only be employed in so-called latching technology circuits, which are slow to switch and subject to "punch-through," or they need to be externally shunted to have nonhysteretic behavior which comes at the cost of reducing I_cR_N. The Josephson effect also occurs in superconductor-normal metal-superconductor (SNS) junctions, where the superconductivity leaks through the normal metal via the proximity effect, and the supercurrent is carried mostly by Andreev bound states localized primarily within the barrier. SNS junctions often have nonhysteretic current-voltage characteristics, which allow for the possibility of rapid single flux quantum (RSFQ) logic [3]. RSFQ logic runs the fastest superconducting circuits, with the optimal speed inversely proportional to I_cR_N (but many circuits are run at less than optimal speed).

Since the resistance of a SNS junction is low (and often the critical current density of a SIS junction is low) it has long been conjectured that one may be able to optimize I_cR_N with a barrier tuned to lie close to the metal-insulator transition (where both I_c and R_N will be moderate) [4]. Production of such superconductor-correlated metal-superconductor (SCmS) junctions would be useful as well, because they would likely be self-shunted and not require an additional shunt resistor to establish a nonhysteretic current-voltage characteristic (as is currently needed with SIS junctions employed in RSFQ logic).

We present evidence here that supports the notion that Josephson properties are optimized with SCmS junctions, where the correlated metal (or insulator) is tuned to lie close to a metal-insulator transition. Successful operation and manufacture of RSFQ circuits requires four important elements: i) a large value of I_cR_N to ensure rapid-speed operation of the circuit; ii) a small temperature derivative of the critical current $dI_c(T)/dT$ to ensure robustness against thermal variations in the circuit; iii) good junction uniformity across the chip; and iv) nonhysteretic current-voltage characteristics to allow RSFQ logic to be used. We will discuss the first three elements here, and defer the last to future work.

II. Results

The description of a correlated metal barrier (and of the metal-insulator transition) lies outside the highly successful realm of quasiclassical approximations to the theory of Josephson junctions [5]. Hence, there is a need to develop a many-body-physics formalism that is capable of describing such correlated electron phenomena. Recently, progress in the application of the dynamical mean field theory to inhomogeneous systems [6], [7], [8] has allowed self-consistent modeling of short-coherence length superconductors attached to correlated metal barriers that form Josephson junctions. The

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only approximation used within the formalism is the so-called local approximation, which says that the electronic self energy possesses no momentum dependence (although it can vary spatially due to the inhomogeneous layered structure of the device). Such an approximation is known to work well for three-dimensional systems.

We describe our Josephson junction by a lattice model with each lattice site corresponding to a unit cell of the superconductor or the barrier material. In our calculations the electrons move on a simple cubic lattice. The superconductor is described by an attractive Hubbard Hamiltonian in the mean-field approximation (which is identical to a BCS approximation except the energy cutoff is determined by the electronic bandwidth, since there are no phonons) [9]. The barrier material is described by the Falicov–Kimball model [10] with dynamical mean-field theory, which is identical to the coherent potential approximation of a binary alloy problem, with the difference in on-site potentials being equal to U_{FK}, and the alloy distribution treated in an annealed fashion (50% concentration of A and B ions). The Hamiltonian for the Josephson junction is

$$H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_i U_i \left(c_{i\uparrow}^{\dagger} c_{i\uparrow} - \tfrac{1}{2} \right) \left(c_{i\downarrow}^{\dagger} c_{i\downarrow} - \tfrac{1}{2} \right) + \sum_{i\sigma} U_i^{FK} c_{i\sigma}^{\dagger} c_{i\sigma} \left(w_i - \tfrac{1}{2} \right), \quad (1)$$

where $c_{i\sigma}^{\dagger}$ ($c_{i\sigma}$) creates (destroys) an electron of spin σ at site i on a simple cubic lattice, $U_i = -2$ is the attractive Hubbard interaction for sites within the superconducting planes (and $U_i = 0$ in the barrier), U_i^{FK} is the Falicov–Kimball interaction which is nonzero for planes within the barrier (which tunes the metallicity of the barrier), and w_i is a classical variable that equals 1 if a disorder ion occupies site i and is zero if no disorder ion occupies site i. A chemical potential μ is employed to determine the filling. The superconductor and barrier are always chosen to be at half filling here ($n = 1$), hence $\mu = 0$. All energies are measured in units of the hopping integral $t = 1$; the electronic bandwidth is 12. We take $U = -2$ in the superconductor, which yields $T_c = 0.112$ and $\Delta(T = 0) = 0.198$. We vary U_{FK} within the barrier to tune the metallicity (with the thermodynamic average of w_i fixed at $1/2$); in the bulk, a metal-insulator transition occurs at $U_{FK} \approx 4.9$.

In our calculations we have a self-consistent region, where the superconducting parameters can vary from plane to plane, and a bulk region, which serves as the infinite superconducting leads for the junction. The bulk superconducting coherence length is approximately 3.7 lattice spacings, so we use 30 planes on either side of the barrier as the self-consistent region (approximately 8 times the coherence length). In this work, we take either one or five planes for the barrier to simulate a thin (tunnel) junction and a moderately thick junction. The equations for the Green's functions are iterated for self consistency of at least one part in 10^5. Once a self-consistent solution is determined, we verify that it maintains current conservation throughout the junction.

Calculations at low temperature $(T_c/11)$ have reproduced [12] the Ambegaokar–Baratoff result of I_cR_N being independent of the size of the insulating gap (or equivalently independent of U_{FK} for large U_{FK}) for a thin tunnel junction and equal to $\pi\Delta/2e$. The calculations also showed that for moderately thick junctions, I_cR_N is strongly enhanced above the Ambegaokar–Baratoff prediction on the insulating side of the transition. Thicker junctions show no such optimization, because the critical current falls faster than the resistance grows in the thick insulating regime. *This implies that by a proper tuning of the thickness of the barrier and its metallicity, optimal I_cR_N values can be achieved.*

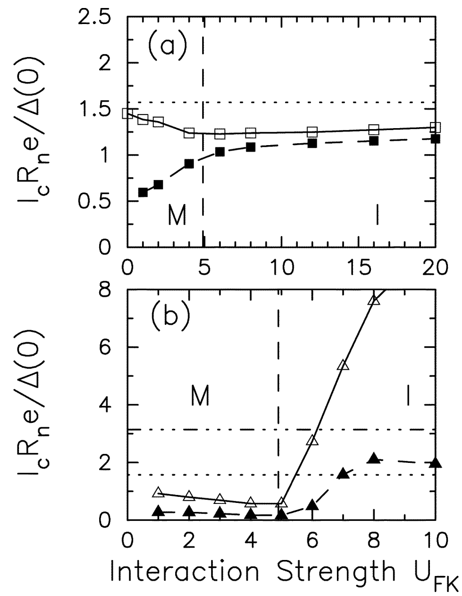

Fig. 1. Characteristic voltage I_cR_N [normalized by $\Delta(0)/e$] for (a) a thin ($N = 1$) barrier and (b) a moderately thick ($N = 5$) barrier at $T \approx T_c/11$ and $T \approx T_c/2$ as a function of the "metallicity" U_{FK}. The bulk metal-insulator transition (vertical dashed line) occurs at $U_{FK} \approx 4.9$; in these junctions the critical U_{FK} is somewhat larger due to their finite extent. The open symbols (and solid line) depict the low-temperature ($T \approx T_c/11$) results and the solid symbols (and dashed line) depict the higher-temperature ($T \approx T_c/2$) results. The dotted line, in both cases is the $T = 0$ Ambegaokar–Baratoff prediction and the chain-dotted line is the Kulik–Omelyanchuk prediction. Note how the single plane results are optimized in the metallic region for low T and in the insulating regime for high T, but the Ambegaokar–Baratoff result is reduced by about 20% due to band-structure effects. In the five-plane case, the characteristic voltage is enhanced on the insulating side of the metal-insulator transition, and the optimization continues to be there for $T_c/2$, but it is reduced both due to a reduction in I_c *and* to a reduction in R_N.

The next issue to tackle is the stability to temperature variations. We show in Fig. 1 the characteristic voltage for a thin single-plane barrier $N = 1$ Josephson junction at low temperature $T_c/11$ and at moderate temperature $T_c/2$ (the low-temperature data are corrected from [12] due to more accurate R_N values). In the low temperature regime, we see an optimization of I_cR_N for clean metallic SNS junctions. The I_cR_N product at $U_{FK} = 0$ is maximized by the product of the critical current density in the bulk times the Sharvin resistance times the cross-sectional area of a unit cell. This product is smaller than the Kulik–Omelyanchuk prediction of $\pi\Delta/e$ [11]. The characteristic voltage drops rapidly with T for such SNS junctions, and the optimization lies with the insulating tunnel junctions for higher temperature. Note that the flatness of the I_cR_N curves indicates the Ambegaokar–Baratoff prediction holds in these results as well (we estimate the error in our calculations to be about the symbol size). Also shown in Fig. 1 are the results for

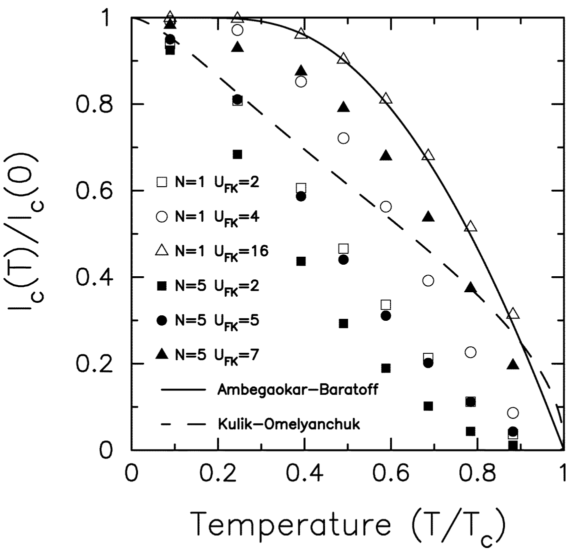

Fig. 2. Temperature dependence of the critical current for a variety of different Josephson junctions. The open symbols are for a single plane $N = 1$ and the solid symbols are for a moderately thick $N = 5$ junction. Each thickness has three different levels of correlation ranging from metallic $U_{FK} = 2$ to intermediate $U_{FK} = 4$ to insulating $U_{FK} = 16$ for the single-plane junction, and metallic $U_{FK} = 2$ to dirty metal $U_{FK} = 5$ to correlated insulator $U_{FK} = 7$ for the $N = 5$ junction. In addition, we include the Ambegaokar–Baratoff analytic result (solid line) and the Kulik–Omelyanchuk clean SNS result (dashed line). Note how the thin insulator reproduces the expected Ambegaokar–Baratoff result, and that the more metallic proximity-effect junctions have much worse temperature dependence and wide linear regimes just like Kulik–Omelyanchuk predict. The moderately thick correlated insulator, seems to have optimized properties.

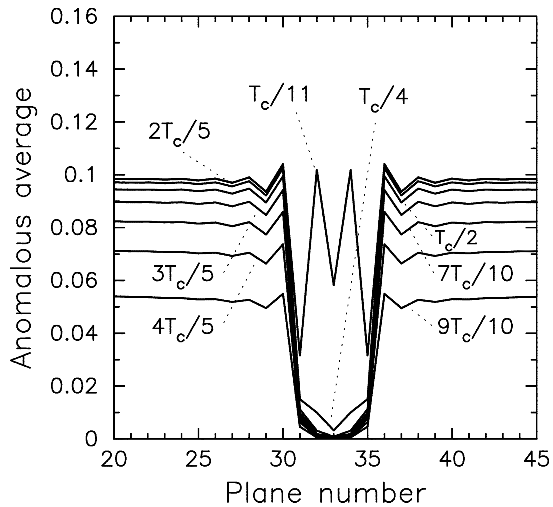

Fig. 3. Anomalous average as a function of plane number for the optimized Josephson junction with $N = 5$ and $U_{FK} = 7$. Note how there are Fermi wavevector oscillations in the superconductor (planes 1–30 and 36–66) as we approach the superconductor-barrier interface, but the large oscillations in the barrier are only present at very low T.

a moderately thick five-plane barrier $N = 5$ Josephson junction. There we see I_cR_N also decreases for the SNS junctions as the scattering increases, but once the metal-insulator transition occurs, I_cR_N jumps at approximately the dashed line, and is strongly enhanced on the insulating side. This optimization continues to hold at moderate temperature as well (although the optimization is less dramatic as T increases). The significant drop in magnitude of I_cR_N arises from a decrease in *both* $I_c(T)$ (which is similar to the Ambegaokar–Baratoff rate, see below) and $R_N(T)$ (which drops due to the effect of correlations at finite T allowing more thermally activated carriers). But the majority of the drop comes from the large decrease in R_N from $T = T_c/11$ to $T = T_c/2$; it is likely the normal-state resistance has an exponential dependence on temperature in this region. This may require circuits to be run at lower temperatures to take advantage of the faster speed.

In order to examine the temperature dependence more thoroughly, we plot $I_c(T)/I_c(0)$ for a variety of different temperatures and barriers in Fig. 2. Included in that figure is the Ambegaokar–Baratoff analytic result (solid line) and the Kulik–Omelyanchuk result (dashed line). The $N = 1$ SIS junction reproduces the known results for thin tunnel junctions, as expected, while the SNS junctions have poor temperature dependence, since I_c drops very rapidly, and lies far below the Ambegaokar–Baratoff prediction for all T. The correlated metal barrier behaves differently though. For a moderately thick barrier (where I_cR_N was optimized), there is a sharper decrease than in the tunnel junctions only for the low temperature regime $T < 0.3T_c$, where it behaves linearly in T. In the operating regime of $0.4T_c < T < 0.7T_c$, we find the curve is parallel to the Ambegaokar–Baratoff result, implying the temperature derivative is as good as is found in tunnel junctions for that range of operating temperature. *Hence, the SCmS junctions have as good a temperature derivative of the critical current as tunnel junctions within the conventional thermal operating range of a circuit.*

One question that remains is whether the additional temperature dependence of the normal-state resistance in the moderately thick correlated insulator will degrade junction performance by reducing the switching speed and causing timing errors in a circuit. It is best to address this question with a nonequilibrium formalism that can properly handle the junction switching characteristics and will be pursued elsewhere. This analysis will also address the issue of junction capacitance and whether SCmS junctions will be hysteretic, requiring external shunts.

The issue of junction uniformity across a chip may also be difficult for SCmS junctions. The reason why this is so is due to the fact that one needs to carefully tune the thickness and the metallicity to achieve optimal properties. As U_{FK} increases in magnitude (more insulating), there can be a sensitive dependence of the characteristic voltage on thickness [12]. The sensitivity can be so strong that variation of the thickness by one atomic plane can create “hot spots” or “dead zones” on the junction, producing a situation similar to Josephson coupling through pinholes, even though the barrier is completely homogeneous, except for its thickness. This implies that the barrier thickness may need to be controlled to high precision in order to achieve good spreads in I_cR_N across a chip. It is also possible that one may find that near optimization, the results are not as sensitive to the thickness. Only when systematic examinations on candidate materials are performed will we know if this will be a problem for SCmS junctions.

We wish to end our theoretical discussion by showing one final result on SCmS junctions. This result does not have any bearing on the optimization properties, but it does show some interesting phenomena, that we do not fully understand. In an earlier work [12], we plotted the pair-field amplitude (equal time pair-field correlation function or the anomalous average) as a function of position within the junction; the pair-field amplitude can also be thought of as the limit of the superconducting gap divided by the interaction strength—while the gap is discontinuous through the junction, the pair-field amplitude is not, giving rise to the proximity effect. We found that at low temperature there were small amplitude oscillations within the superconductor as the interface with an insulating barrier was approached. Those oscillations became large in amplitude within the barrier and were localized at the two interfaces. Here we plot in Fig. 3 the results for the pair-field amplitude as a function of temperature for an $N = 5$ barrier and $U_{FK} = 7$, which is an enhanced case we examined above. One can see that the Fermi wavelength oscillations within the superconductor survive up to high temperature, but the large-amplitude oscillations within the barrier disappear very rapidly as a function of temperature. We do not have any simple explanation for why this occurs.

III. Conclusions

We conclude by summarizing the experimental situation for SCmS junctions. Recent work by Newman, Rowell, and Van Duzer [13] has explored making Josephson junctions out of NbTiN superconductors, with tantalum-deficient Ta_xN barriers. The Ta_xN material has a metal-insulator transition at $x = 0.6$ due to the formation of a gap in the density of states. They are able to grow high-quality junctions with 40 nm wide barriers. These junctions have I_cR_N larger than 1 meV and nonhysteretic current-voltage characteristics. But the thermal properties suffer from a large temperature gradient of the critical current, and the junctions only operate at low temperature (less than $T_c/3$). We believe that the temperature dependence of the critical current will improve by reducing the metallicity within the barrier, and that optimization is certainly possible within these SCmS junctions. But, due to the temperature dependence of the normal-state resistance, circuits may be forced to operate at lower temperatures than $0.4T_c$. A systematic study of Ta_xN junctions that vary the barrier thickness, Ta concentration, and temperature should allow the properties to be optimized.

In conclusion, we have examined a new class of Josephson junctions, the so-called SCmS junctions, in which the barrier is tuned to lie close to a metal-insulator transition. We find that properly optimized junctions in this class show promise as having the highest I_cR_N products and robust temperature dependence for the critical current. Depending on how sensitive the optimization is to the barrier thickness, one may encounter difficulty with characteristic voltage spreads on a chip, but it seems possible that such issues could be resolved with careful engineering. We still need to determine whether these junctions are self-shunted and display nonhysteretic current-voltage characteristics and determine how the temperature dependence of the normal-state resistance and the junction capacitance affects the switching speed. Such analyses require a nonequilibrium formulation, which is currently underway. It is our hope that utilization of SCmS junctions in superconducting electronics will provide further advantages over semiconductor-based technologies and be adopted for some applications in the near term.

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