

Vertex-Corrected Tunneling Inversion in Superconductors*

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We have generalized the McMillan-Rowell tunneling-inversion program, which extracts the electron-phonon spectral function $\alpha^2 F(\Omega)$ and the Coulomb pseudopotential μ^* from experimental tunneling data, to include the lowest-order effect of vertex corrections. In so doing, the momentum dependence of the electron-phonon matrix elements are neglected, which is equivalent to using a local approximation. The vertex-corrected strong-coupling perturbation theory is performed on the imaginary axis and then an exact analytic continuation is used to produce the density of states on the real axis. Comparison with the experimental data for Pb indicates that effects of vertex corrections may be observable even in low-temperature superconductors.

1. INTRODUCTION

The conventional theory of superconductivity is considered to be one of the most well-established theories in condensed matter physics with agreement between theory and experiment being on the order of a few percent or less. A key assumption in this theory is that there is a small parameter, the ratio of the characteristic phonon frequency to the Fermi energy that allows one to neglect higher-order terms (vertex corrections) in the electron self-energy. This is commonly known as Migdal's theorem[1] which was formulated for the superconducting state by Eliashberg[2] to give the standard Migdal-Eliashberg (ME) theory of superconductivity. While this approximation appears to be very successful for low-temperature superconductors, the recent discovery of potentially electron-phonon-mediated high T_c superconductors such as BKBO[3] and the doped fullerenes[4], with high phonon energies and small bandwidths, motivates an investigation of the effect of vertex corrections. Indeed we find that even in the low T_c superconductors, the effects of vertex corrections should be observable.

While there now exists several works which have examined the effect of vertex corrections on T_c [5, 6, 7, 8], here we focus on the tunneling density of states.

We have generalized the McMillan-Rowell tunneling inversion procedure[9], which is used to extract the electron-phonon spectral density $\alpha^2 F(\Omega)$ and the Coulomb pseudopotential μ^* from the experimental tunneling data, to include the lowest-order effect of vertex corrections to the self-energy as shown in Fig. 1. We use the usual local approximation which neglects the momentum dependence in the electron-phonon interaction[5]. The resulting equations are complicated and will be reported elsewhere, here we report our findings for the low-temperature superconductor Pb.

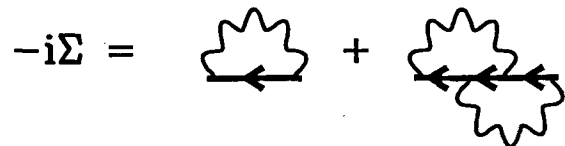


Figure 1. Diagrammatic expansion for the electronic self-energy including the first-order diagram of Migdal-Eliashberg theory and the lowest-order vertex correction.

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2. RESULTS

Although Pb is one of our best-studied strong coupling superconductors, small discrepancies remain between the μ^* required to adjust the calculated bulk T_c to the experimental one and the μ^* extracted along with the $\alpha^2F(\Omega)$ from the tunneling data[10]. As this fitted parameter will change with the inclusion of vertex corrections, it motivated us to examine the case of Pb in developing the vertex-corrected tunneling-inversion procedure. For this purpose, we have included the results of a full bandstructure calculation for the electronic density of states rather than use the free-electron value[6] and have performed a Fermi-surface average for the vertex-corrected terms.

Our results for Pb are found in Fig. 2. In Fig. 2a, we show the extracted $\alpha^2F(\Omega)$ for the vertex-corrected theory. On the scale of this graph, it is hard to distinguish between the vertex-corrected $\alpha^2F(\Omega)$ and the ME one, the main differences being that the vertex-corrections enhance the phonon peaks and reduce the anomalous tail beyond the upper bulk phonon frequency of 9 meV, which usually has been ascribed to surface phonons. In Fig.2b we have plotted the difference between the extracted $\alpha^2F(\Omega)$ for the vertex corrected theory and the one for ordinary ME theory. Here one can clearly see the effects just mentioned. In particular, the enhancement of the peaks in the $\alpha^2F(\Omega)$ by vertex corrections is seen to be on the order of a 1% effect. Hence, even for Pb, the effect of vertex corrections could be observable.

3. CONCLUSIONS

We have developed a tunneling inversion procedure based on the McMillan-Rowell procedure, which includes the lowest-order vertex corrections to the standard ME theory, and we find that the extracted $\alpha^2F(\Omega)$, even for a superconductor such as Pb, will have significant deviations from the ME form. This procedure can be applied to other materials where the deviations are expected to be larger.

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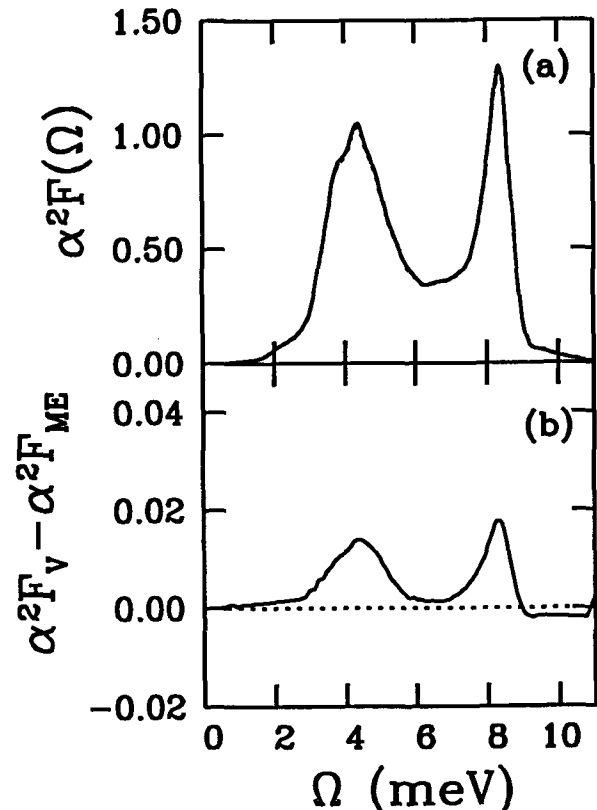


Figure 2. (a) Electron-phonon spectral function, $\alpha^2F(\Omega)$, extracted from the experimental tunneling data for Pb. The solid line is the vertex-corrected fit and the dotted line is the Migdal-Eliashberg fit. (b) The difference between the $\alpha^2F(\Omega)$ with vertex-corrections (V) and that without (ME).