

Vertex Corrections to the Theory of Superconductivity*

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With the eventual goal of understanding vertex corrections in the doped fullerenes and the bismuthates, assuming that they may be electron-phonon superconductors, we have constructed a weak coupling perturbation theory that includes the lowest-order vertex corrections (and reduces to Migdal-Eliashberg theory in the absence of the vertex corrections). Vertex corrections are studied in the local approximation which neglects the momentum dependence of the self energy. This approximation is exact in the limit of infinite dimensions and appears to be accurate in three dimensions. We present results for the superconducting transition temperature as a function of ω_D/E_f and λ and make comparison with similar results calculated in the infinite-dimensional Holstein model in the weak coupling limit. We find that vertex corrections reduce T_c and the energy gap Δ_0 ; however, the ratio $2\Delta_0/k_B T_c$ remains unaffected. We also find that the isotope effect is reduced in the presence of vertex corrections.

The mechanism for superconductivity in the bismuthates and doped fullerenes is not yet established; however, several experiments give evidence in favour of the electron-phonon interaction. To fully investigate this possibility, it is necessary to establish whether the conventional Migdal-Eliashberg theory[1] is applicable in its present form. This theory is based on the approximation, known as Migdal's Theorem[1], that the vertex corrections to the electron self-energy are reduced by the order of ω_D/E_f and hence can be neglected. However, in the doped fullerenes and materials like $Ba_{1-x}K_xBiO_3$ (BKBO), where the electron-phonon interaction is suspected, this ratio is no longer small.

In Fig. 1, we show the diagrammatic expansion for the electronic self-energy which forms the basis of our calculation. The first term is the standard Migdal-Eliashberg approximation and the second term is the lowest-order vertex correction. These diagrams were calculated in the local approximation which neglects the momentum dependence of the self-energy[2]. Details of this calculation will be reported elsewhere. Here we focus on our results. Several recent papers have examined the issue of vertex corrections for specific models[2-5].

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In Fig. 2(a), we show the effect of vertex corrections on T_c , including strong coupling effects. For simplicity we employ an Einstein phonon of energy $\omega_E = 10$ meV and vary its strength to vary $\lambda = 2A/\omega_E$. The coulomb pseudopotential μ^* was set to zero. Along the x-axis, we plot the single spin density of states at fermi energy $\rho(0)$ multiplied by ω_E ; this is essentially the same as the parameter ω_D/E_f , the former being somewhat more universal and less ambiguous. We notice that T_c is reduced with increasing amount of vertex corrections. In the case of the weak coupling limit ($\lambda = 0$), the reduction in T_c from its value without vertex corrections, T_{c0} , is given by the simple formula:

$$T_c/T_{c0} = \exp[-1.25\pi^2\rho(0)\omega_E] \quad (1)$$

Strong coupling reduces the effect of vertex corrections at low phonon frequency but enhances the effect at high frequency.

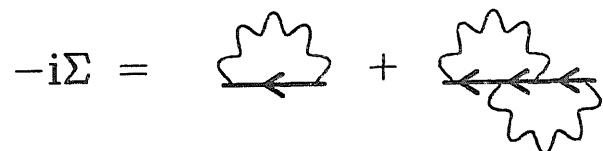


Figure 1. Diagrammatic expansion for the electronic self-energy including the first-order diagram of Migdal-Eliashberg theory and the lowest-order vertex correction.

In Fig. 2(b), we show the effect of vertex corrections on the isotope effect, where the isotope effect coefficient is defined as $\alpha \equiv -d \ln T_c / d \ln M$. Again a simple analytic expression can be obtained in the weak coupling limit:

$$\alpha = 0.5[1 - 1.25\pi^2\rho(0)\omega_E] \quad (2)$$

which is plotted as the solid curve in the graph. Generally, α is suppressed by increasing level of vertex corrections. Strong coupling causes a substantial suppression of α at high frequency but a slightly reduced suppression at low frequency compared with the weak coupling limit. Similar results have also been found recently by others[5].

In Fig. 3, we show results for the infinite-d Holstein model in the weak coupling limit[4]. The x-axis is an effective ω_D/E_f -type parameter. Shown are T_c curves for various filling (the energy-dependent density of states (DOS) is a gaussian in infinite-d with an effective bandwidth of $4t^*$). At half-filling, we have the same result as our $\lambda = 0$ result of Fig. 2(a) confirming our work in three dimensions. Also shown are curves for various filling with the interesting feature that at low doping, vertex corrections enhance T_c . We expect that when we include an energy-dependent DOS and particle-hole asymmetry in our three dimensional calculation, we will find similar results. For BKBO, this could be a plausible explanation for its high T_c as it is doped far away from half-filling. The doped fullerenes are near half-filling and so this enhancement effect is not likely to be relevant for explaining high T_c 's in these materials.

Our conclusions are summarized in the abstract.

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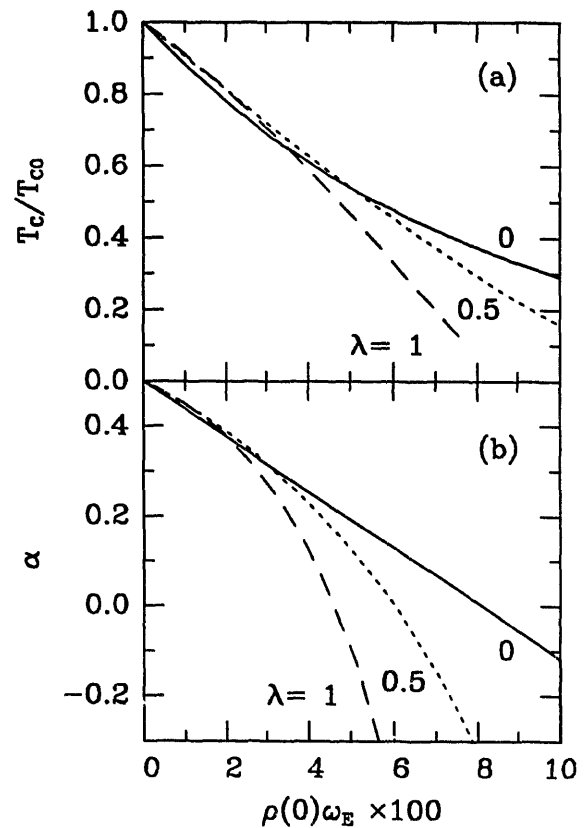


Figure 2. (a) Reduction in T_c with vertex corrections from the T_{c0} without vertex corrections as a function of $\rho(0)\omega_E$ for different amounts of strong coupling. (b) The reduction in the isotope effect coefficient α with vertex corrections.

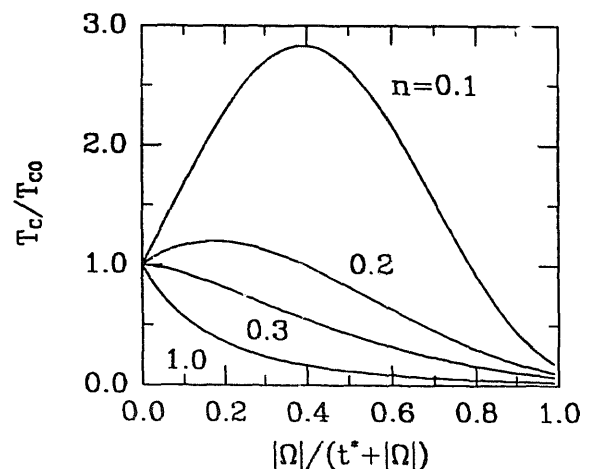


Figure 3. Change in T_c with vertex corrections in the infinite-d Holstein model (see text). Curves are plotted for different electron density n , from half-filled (bottom) to very low doping (top).