Tutorial 2: Probability

1. Coin Tosses

Recall that the probability of a specified outcome or outcome(s) is given by

$$\text{prob. of success } = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}. \tag{1}$$

We will use this rule to determine the probability of various outcomes when three coins are tossed.

(a) How many total possible outcomes exist?

(b) How many outcomes have zero tails? one tail? two tails? three tails?

(c) Fill in the following table for the probability of seeing 0, 1, 2, or 3 tails.

Number of tails:	0	1	2	3
Probability:				

(d) Each group should perform the experiment (3 coin tosses) eight times. Record your group's results below:

Trial Number	Result (e.g. HHT)	Number of tails
1		
2		
3		
4		
5		
6		
7		
8		

(e) How many times did your group get two tails? How many times did you get zero tails? Do your results match up with the probabilities you determined in (c)? If so, would you expect this for every 8 times you do the experiment? If not, explain why there might be a difference.

(f) Add your results to the table on the blackboard. What fraction of the time did the entire class get two tails? Zero tails? How does this compare to the probabilities in (c)?

Two students, Daria and Quinn, are having a discussion about probability:

- *Daria*: I am confused by these probabilities, if the probability of finding two heads on three coin tosses is 3/8, then I should always see two heads appearing three times on each group's results, but that did not occur. Some of the groups must have made up their results.
- *Quinn*: No Daria, the experiment has to be repeated many many times before the probability can be predicted accurately by the experiment. If we tossed the coins 80 times we would most likely see two heads come up 29 or 30 or 31 times, rather than 10 times, or 50 times. Let's go to the mall.

With which student do you agree ? Explain your reasoning.

How many times must an experiment be performed before you can accurately predict the probabilities of the different outcomes? This is an important question that was initially studied by an employee of the Guinness Brewery (yes, the same place that makes Guinness Stout) named Gossett. Gossett's work on this problem in the early 1900's is one of the first examples of industrial research in basic science that resulted in a competitive edge for the company. In fact, Gossett was allowed to publish his results to the scientific community, but he had to use a pseudonym, so that other breweries would not realize that Guinness had a competitive advantage!

2. Compound probabilities

Suppose we roll two dice. What's the probability of rolling a total of 4 **or** 11?

(a) First let's apply the general definition for probabilities (Eq. 1). How many possible outcomes are there for rolling two dice?

How many of these outcomes correspond to a 4? How many to an 11? How many to a 4 **or** 11?

What is the probability of rolling a 4 **or** 11?

(b) Explain how you would compute the probability using the rules of compound probability discussed in Section 5.2 of *Strange World*. Verify that these rules yield the same probability as you found in (a).

Suppose we roll two dice, one green and one red. What's the probability of rolling either a 2 on the red die **or** a 4 on the red die **and** a rolling a 5 on the green one?

(a) First let's apply the general definition for probabilities (Eq. 1). How many possible outcomes are there for rolling the two dice?

How many of these outcomes are successful?

What is the probability of a successful outcome?

(b) Explain how you would compute the probability using the rules of compound probability discussed in Section 5.2 of *Strange World*. Verify that these rules yield the same probability as you found in (a).

3. Birthdays

What is the probability that in a room of 30 people, at least two people have the same birthday? The answer is 71%! To most people, this answer seems surprisingly large. After all, there are 365 possible birthdays. It doesn't seem very likely that two people out of 30 should have the same birthday.
Let's consider a similar, but simpler problem. What is the probability that in a group of six people, at least two have birthdays in the same month? (Make a guess before going through the calculation.)
Note that this problem is quite challenging. If you are having trouble figuring out how to proceed, consult an instructor.

(a) What's the probability that in a group of two people both have birthdays in the same month?

(b) What's the probability that the two people (from a group of two) **do not** have birthdays in the same month?

(c) What is the probability that two people have birthdays in the same month **or** two people do not have birthdays in the same month? Use the rules for compound probabilities to determine your result. Explain why you got the answer you did. This result is important for part (e) below.

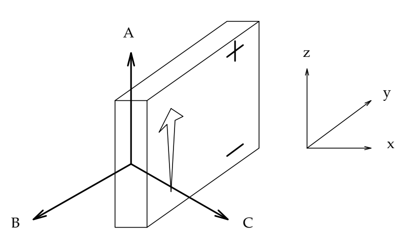
(d) In a group of three people, what's the probability that none of them have birthdays in the same month? (Hint: one way to rephrase this is as follows: what is the probability that the first two do not have birthdays in the same month **and** the third person's birthday is in *yet another month*?)

(e) In a group of three people, what's the probability that at least two of them have birthdays in the same month? (Hint: consider your answer for part (c) above)

(f) For a group of six people, write an expression for the probability that at least two people have birthdays in the same month. (You'll need a calculator to evaluate the expression.)

(g) Write an expression for the probability that at least two people in a group of 30 will have the same birthday (only try to evaluate this if you are brave).

4. Rotating Stern-Gerlach Experiment



A Stern-Gerlach analyzer is mounted so that it can be rotated to have a magnetic field point in any of three directions: **A**, **B**, or **C**. The analyzer is switched *at random* between these three orientations, with each orientation having a probability of $\frac{1}{3}$. An atom is injected with $m_z = +m_B$ into the rotating analyzer. The goal of this problem is to find the probability that it will leave the + exit by completing the following steps:

(a) Suppose the analyzer is not rotating, but remains fixed in one of it's orientations. For each of the orientations **A**, **B**, and **C**, determine the probability that the atom leaves from the + exit. *Hint*: recall that the probability that an atom emerges from the positive exit of an analyzer tilted at an angle θ is $\cos^2(\frac{\theta}{2})$ and that $\cos^2(\frac{120^\circ}{2}) = \frac{1}{4}$.

(b) Describe in words (using **and** & **or**) what the total probability is for the atom to emerge from the + exit, assuming the analyzer is rotating, and points at either **A**, **B**, or **C**. Hint: your answer should depend on the probability that the analyzer is located at orientation **A**, ...

(c) Determine the total probability for the atom to emerge from the + exit assuming that the analyzer switches at random between these three orientations, with each orientation having a probability of 1/3.

(d) What is the probability the atom will emerge from the $-$ exit?

(e) Suppose the analyzer breaks, so it no longer stops at **C**, but still stops at random at **A** and **B** (each with probability 1/2). Calculate the probability an atom will emerge from the + and - exits now.