1.) A system with degenerate eigenstates that is subjected to a time-independent perturbation is being driven at its resonance frequency ($\omega = 0$). Consider the following example.

A Hydrogen atom in its ground state is prepared for $t \le 0$ with spin up along $\hat{z}$. At time $t=0$ a constant B field is turned on which points in an arbitrary direction $(\theta, \phi)$. Neglect fine structure (and $\vec{A}^2$ terms). Compute the probability that the atom will be found in the ground state with spin down as a function of time. Do the problem first with first-order perturbation theory and then solve it exactly. Discuss the accuracy of the perturbation theory.

2.) A "nailed-down" spin-$\frac{1}{2}$ particle is acted on by a constant magnetic field in the $z$ direction and by an oscillating field in the x-y plane.

$$\hat{H} = \hat{H}_0 + \hat{V}(t) \qquad \hat{H}_0 = \hbar \Omega_0 \hat{S}_z \qquad \hat{V}(t) = \hbar \Omega_1 (\cos\omega t \, \hat{S}_x + \sin\omega t \, \hat{S}_y)$$

a.) at $t=0$ the particle is in the spin-up state along the z-axis. What is the probability it will be found up at time t? (Solve the problem exactly)

b.) Use time-dependent perturbation theory to <u>second-order</u> to compute the probability.

c.) Compare the perturbation theory to the exact result expanded to second order. Comment on the accuracy of the perturbation theory.

3.) Time-ordered product gymnastics

Consider the time-dependent harmonic oscillator

$$\hat{H}(t) = \hat{H}_0 + \hat{V}(t)$$

$$\hat{H}_0 = \hbar\omega\,(a^\dagger a + \tfrac{1}{2}) \qquad \hat{V}(t) = c\left(e^{i\Omega t}\hat{a}^\dagger + e^{-i\Omega t}\hat{a}\right)$$

a.) Compute $\hat{U}_S(t,0)$ from the interaction representation formula

$$\hat{U}_S(t,0) = e^{-\frac{i}{\hbar}\hat{H}_0 t}\, T\, e^{-\frac{i}{\hbar}\int_0^t dt'\, \hat{V}_I(t')}$$

to second order in $\hat{V}$.

b.) Compute $\hat{U}_S(t,0)$ from the formal solution of Schroedinger's equation

$$U_S(t,0) = T\, e^{-\frac{i}{\hbar}\int_0^t dt'\, \hat{H}(t')}$$

to second order in $\hat{H}$,

c.) Compare the two results and comment on the differences.