PHYS 5002: Homework 10

1 Ion trap problem

Consider the three ion problem. Let

$$\hat{H} = \sum_{ij=1}^{3} J_{ij}\sigma_i^x \sigma_j^x + B(t)\sum_{i=1}^{3}\sigma_i^y$$

and use the symmetrized states discussed in the lectures. Let

$$J_{ij} = -\frac{\hbar}{2}\sum_{\alpha=1}^{N}\Omega_i\Omega_j b_i^\alpha b_j^\alpha \frac{(\delta k)^2\hbar}{2m}\frac{1}{\omega_\alpha^2 - \mu^2}$$

by neglecting the time-dependent terms. For the experiment, assume $\Omega_i = \Omega_j$. Use the b_i^α's derived in class.

a.) There are two important J_{ij} values

$$J_{12} = J_{21} = J_{13} = J_{31} = J$$
$$J_{13} = J_{31} = J'.$$

Let $|J| = 2\pi \cdot 800$ Hz. Find J' in the same units under the following assumptions:

$$\frac{\omega_{tilt}}{\omega_{com}} = \frac{2.5474}{2.7367} \text{ and } \frac{\omega_{zig-zag}}{\omega_{com}} = \frac{2.2560}{2.7367}$$

with $\mu = 0.874 \cdot \omega_{com}$.

<u>HINT:</u> Compute J/J' and use the J value given above after you determine the sign of J. Your resultant Hamiltonian should look like

$$\hat{H} = J(\sigma_1^x\sigma_2^x + \sigma_2^x\sigma_3^x) + J'\sigma_1^x\sigma_3^x + B(t)(\sigma_1^y + \sigma_2^y + \sigma_3^y)$$

b.) Using the symmetrized states, compute $H(t)$ in each symmetry sector. Find which sector the ground state lies in for all B values.

c.) Assume the adiabatic approximation—the state starts polarized along the y-axis for each ion site. At any later time, we follow the ground state at the given value of B. Pick $B = -B_0\exp(-t/\tau)$ and

$$B_0 = 2\pi \cdot 10,000 \text{ Hz}$$
$$\tau = 10^{-4} \text{ sec}$$

Run your simulation out to $t = 0.5\times10^{-3}$ s. Let $P(\text{state})$ denote the probability to be in a given state. Plot $P(|\uparrow\uparrow\uparrow\rangle_x) + P(|\downarrow\downarrow\downarrow\rangle_x)$, $P(|\uparrow\downarrow\uparrow\rangle_x) + P(|\downarrow\uparrow\downarrow\rangle_x)$, and $P(|\uparrow\uparrow\downarrow\rangle_x) + P(|\downarrow\uparrow\uparrow\rangle) + P(|\downarrow\downarrow\uparrow\rangle) + P(|\uparrow\downarrow\downarrow\rangle)$ versus time on the same plot.

d.) Repeat (c) using the sudden approximation. To do this, we start the state initially polarized along the y-axis and compute the three overlaps:

$$\alpha = \langle\psi_{gs}|\psi_{pol}\rangle$$
$$\beta = \langle\psi_{1st\ ex}|\psi_{pol}\rangle$$
$$\gamma = \langle\psi_{2nd\ ex}|\psi_{pol}\rangle$$

where $|\psi_{pol}\rangle$ is the state polarized along the y-axis, and $|\psi_{gs}\rangle$, $|\psi_{1st\ ex}\rangle$, and $|\psi_{2nd\ ex}\rangle$ are the ground state and first two excited states of the system *in the symmetry sector where the ground state lies.* Then at any time later, the sudden approximation says

$$|\psi(t)\rangle_{sudden} = \alpha\,|\psi_{gs}(t)\rangle + \beta\,|\psi_{1st}(t)\rangle + \gamma\,|\psi_{2nd}(t)\rangle\,.$$

Use this to repeat the same calculation as above and produce the relevant probability plots.

2 Shake-off excitation

Assume a spherical well of radius R:

$$V(r) = \begin{cases} 0 & 0 \le r < R \\ \infty & r > R \end{cases}$$

The s-wave bound states are $|n\rangle = \frac{A}{r}\sin\frac{n\pi r}{R}$ and A is found through normalization. If an electron is in the ground state and R suddenly increases to $R' = R/\nu$ for $0 \le \nu \le 1$...

a.) What is the probability to find the electron in the new ground state? (plot as a function of ν)

b.) What is the probability that the electron will be in an excited state with energy n?

c.) Sum the probabilities to show that

$$\sum_{n'} P_{n\leftarrow 1} = 1$$

for the case $\nu = \frac{1}{2}$.

3 Beta decay

A tritium nucleus ($Z = 1$) undergoes beta decay, where a neutron in the nucleus emits an electron and changes to a proton ($Z = 2$). The emitted electron receives so much energy that it is not bound to the ion and flies away almost instantly.

a.) If the electron orbiting the tritium atom is in the ground state, what is the probability that it lies in the state with principal quantum number n in the Helium ion?

b.) Sum up the probability

$$\sum_{n=1}^{\infty} P_{n\leftarrow gs}$$

Note that this sum is <u>not</u> equal to one. Explain why. Note that you have to do this sum numerically.