PHYS 5002: Homework 11

1 Photoproduction of hydrogen

Using the notation $|j, m_j\rangle$ we have

$$\begin{aligned}
\left|2p_{3/2}, 3/2\right\rangle &= \psi_{211}(r)\left|\uparrow\right\rangle \\
\left|2p_{3/2}, 1/2\right\rangle &= \sqrt{\frac{2}{3}}\psi_{210}(r)\left|\uparrow\right\rangle + \sqrt{\frac{1}{3}}\psi_{211}(r)\left|\downarrow\right\rangle \\
\left|2p_{3/2}, -1/2\right\rangle &= \sqrt{\frac{2}{3}}\psi_{210}(r)\left|\downarrow\right\rangle + \sqrt{\frac{1}{3}}\psi_{21-1}(r)\left|\uparrow\right\rangle \\
\left|2p_{3/2}, -3/2\right\rangle &= \psi_{21-1}(r)\left|\downarrow\right\rangle
\end{aligned}$$

$$\begin{aligned}
\left|2p_{1/2}, 1/2\right\rangle &= \sqrt{\frac{1}{3}}\psi_{210}(r)\left|\uparrow\right\rangle - \sqrt{\frac{2}{3}}\psi_{211}(r)\left|\downarrow\right\rangle \\
\left|2p_{1/2}, -1/2\right\rangle &= \sqrt{\frac{1}{3}}\psi_{210}(r)\left|\downarrow\right\rangle - \sqrt{\frac{2}{3}}\psi_{21-1}(r)\left|\uparrow\right\rangle
\end{aligned}$$

$$\begin{aligned}
\left|2s_{1/2}, 1/2\right\rangle &= \psi_{200}(r)\left|\uparrow\right\rangle \\
\left|2s_{1/2}, -1/2\right\rangle &= \psi_{200}(r)\left|\downarrow\right\rangle
\end{aligned}$$

These are all of the degenerate $n = 2$ levels. Recall the transition rate $\Gamma_{f\leftarrow i}$ satisfies

$$\Gamma_{f\leftarrow i} = \frac{4\omega^3 e^2}{3\hbar c^3}\left|\left\langle\psi_f|\mathbf{r}|\psi_i\right\rangle\right|^2$$

in the dipole approximation. Also, recall that the lifetime of states satisfies

$$\tau_{f\leftarrow i} = \frac{1}{\Gamma_{f\leftarrow i}}$$

Calculate $\tau_{1s_{1/2}\leftarrow 2p_{3/2}}, \tau_{1s_{1/2}\leftarrow 2p_{1/2}}$, and $\tau_{1s_{1/2}\leftarrow 2s_{1/2}}$ in the dipole approximation for each m_j value. Express your final answer in seconds.

2 Final degenerate perturbation theory problem

Consider a five state system with the following $\hat{H}$:

$$\hat{H}_0 = \begin{pmatrix} E_0^0 & 0 & 0 & 0 & 0 \\ 0 & E_0^0 & 0 & 0 & 0 \\ 0 & 0 & E_0^0 & 0 & 0 \\ 0 & 0 & 0 & E_0^0 & 0 \\ 0 & 0 & 0 & 0 & E_0^1 \end{pmatrix}, \ \hat{V} = \begin{pmatrix} 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & a & d \\ 0 & 0 & a & 0 & e \\ b & c & d & e & f \end{pmatrix}$$

with a, b, c, d, e, f all real. This has four degenerate and one nondegenerate level for $\hat{H}_0$. Compute all energies to order V^2. Note the degeneracy is only partially lifted to first order.